Probing internal dissipative processes of neutron stars with gravitational waves

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Justin Ripley, Abhishek Hegade K.R., Nicolás Yunes
University of Illinois Urbana-Champaign & ICASU

arXiv:2306.15633

What is the equation of state of high-density nuclear matter?

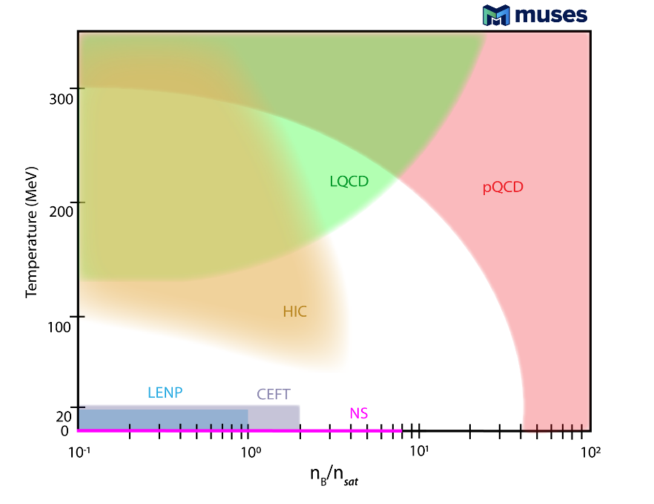

Figure 1: MUSES collaboration 2303.17021

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi$$

What is the equation of state of high-density nuclear matter?

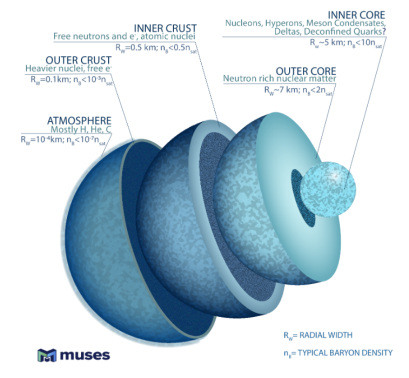

Figure 2: MUSES collaboration 2303.17021

- Neutron stars: densest objects in the universe
  $\rho \sim 10^{14-15}\mathrm{g}/\mathrm{cm}^3 \gtrsim n_{sat}$

Neutron star (NS) equation of state (EOS)

- Model neutron star with relativistic, perfect fluid stress-energy tensor

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu}.$$

- What is the equation of state $p(\rho)$?

Mass vs. radius

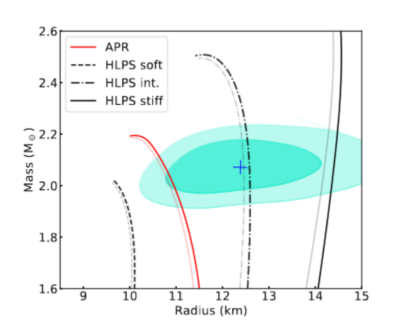

Figure 3: Riley+ 2105.06980

$$\frac{dP}{dr} = -\frac{\left(m + 4\pi r^3 P\right)\left(\rho + P\right)}{r\left(r - 2m\right)}, \qquad \frac{dm}{dr} = 4\pi r^2 \rho.$$

- M (and/or R) $\implies$ constraint on $p\left(\rho\right)$

Tidal deformability

$$Q_{ij} \sim -\Lambda E_{ij}$$

- $\Lambda\,(p,\rho)$ (Flanagan+Hinderer 2007, Hinderer 2007, Damour+Nagar 2009, Binnington+Poisson 2009)

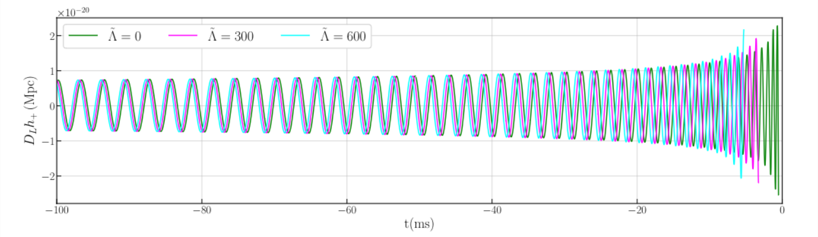

Figure 4: Chatziioannou 2006.03168

Tidal deformability

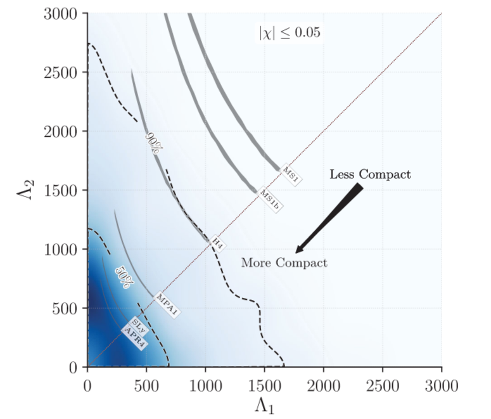

Figure 5: LIGO/Virgo 1710.05832

- GW phase $\implies$ constraint on Λ $\implies$ constraint on $p(\rho)$

Out-of-equilibrium physics of neutron stars 1

- Viscous (higher derivative) corrections

$$T_{\mu\nu} = \left(\rho + p + \boxed{\zeta\theta}\right) u_\mu u_\nu + p g_{\mu\nu} - \boxed{2\eta\sigma_{\mu\nu}} + \cdots.$$

  - ζ: Urca processes $n \to p + e^- + \bar{\nu}_e$, $p \to e^- \to n + \nu_e$ (Haensel 2002, Alford+ 2018, Most+ 2021)
  - η: NS crust dynamics (e.g. Kochaneck 1992)
- What is $\zeta(\rho)$ and $\eta(\rho)$?

Out-of-equilibrium physics of neutron stars 2

- Merger: out-of-equilibrium properties of the star could be measurable with GWs as remnant star relaxes to equilibrium (Alford+ 2018, Most+ 2021, 2022, Rezzolla+ 2023).

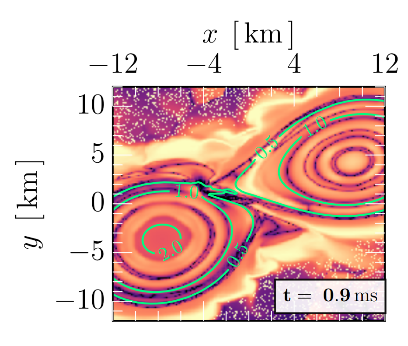

Figure 6: ln (Π/ρ), Most+ arXiv:2207.00442

Out-of-equilibrium physics of neutron stars 3

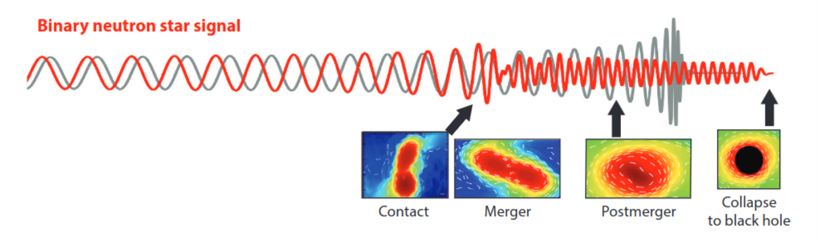

Figure 7: Lattimer, Annu. Rev. Nucl. Part. Sci. 2021. 71:433–64

- The merger/postmerger is messy
- Constrain out-of-equilibrium effects during the inspiral?

Previous work

- Dynamical tides (modal excitation): important **tidal resonance/very rapid rotation** (Lai 1994, Ho+Lai 1999, Lai+Wu 2006, Flanagan+Racine 2007, Hinderer+ 2016, ...)
  - May also be important not during resonance (Pratten+ 2022)
- Viscous effects and tidal locking (Bildsten+Cutler 1992)
- Viscous effects and change in NS shape (Kochaneck 1992)
- Dissipation: 4PN order in the GW phase (Poisson+Sasaki 1994, Tagoshi+ 1997, Alvi 2001)
- Specific PN contribution of bulk viscosity during inspiral not measurable (Most+ 2021)

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Projecting the likely importance of weak-interaction-driven bulk viscosity in neutron star mergers

Elias R. Most,[1,2,3★] Steven P. Harris,[4★] Christopher Plumberg,[5] Mark G. Alford,[6] Jorge Noronha,[5★] Jacquelyn Noronha-Hostler,[5] Frans Pretorius,[2,7] Helvi Witek[5] and Nicolás Yunes[5]

Main conclusions

$$Q_{ij} \sim -\Lambda E_{ij} - \Xi \partial_t E_{ij}.$$

- **Dissipative tidal deformability** Ξ: correction to GW phase from dissipative (e.g. viscous) processes
- Potentially similar contribution to GW phase as Λ if $\tau_d \gtrsim 20\mu\mathrm{s}$
- Dissipative/viscous effects enter the GW phase at 4 Post-Newtonian (PN) relative order, with a large finite-size correction

$$h(f) = A(f)\, e^{i\Psi(f)},$$
$$\Psi_{\Xi}(f) = -\frac{75}{512}\frac{1}{\eta}\bar{\Xi} \times (\pi M f)\log(\pi M f).$$

- **Physically allowed values of viscosity may be measurable with ground-based GW detectors**

Overview of rest of talk

1. Computing the GW phase
2. Tidal response of a relativistic star
3. Post-Newtonian (PN) expansion of neutron star binaries
4. Detectability
5. Calculating Ξ for (non)relativistic stars

Computing the GW phase

- GW strain h

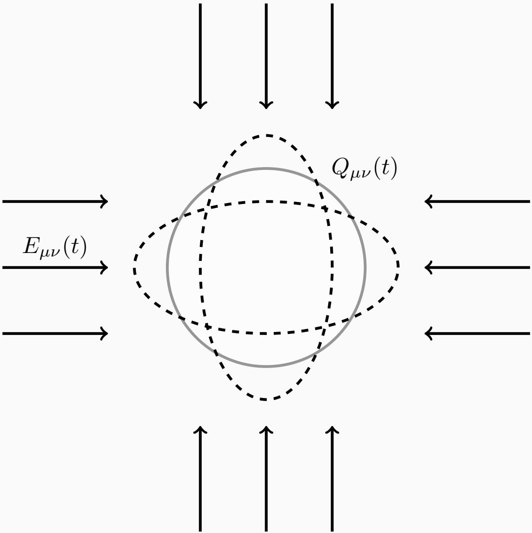
$$h(f) = A(f)\, e^{i\Psi(f)}.$$

- Differential equation for phase in terms of total binding energy E_{tot} of a binary (Tichy+ 2000)

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

- Compute $E_{tot}, \dot{E}_{tot}$ for a Newtonian binary, including keeping dissipative tidal response

Tidal response of a star



Tidal response of a star

- Assume slowly changing tidal field

$$\frac{dE^{\mu\nu}}{d\tau} \ll \omega_f E_{\mu\nu}, \quad \frac{d^2 E^{\mu\nu}}{d\tau^2} \ll \omega_f \frac{dE^{\mu\nu}}{d\tau}, \cdots$$

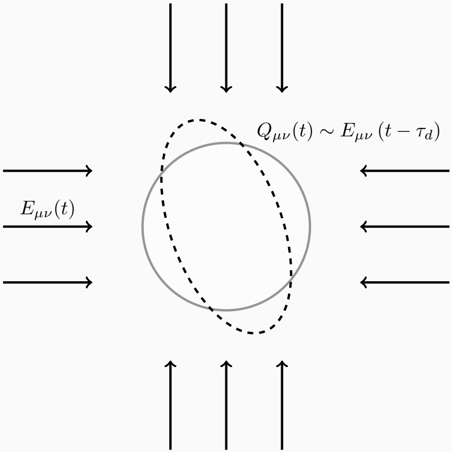
- Assume linear quadrupolar reponse

$$Q^{\mu\nu} = -\lambda_2 E^{\mu\nu} - \boxed{\lambda_2 \tau_2^{(1)} \frac{dE^{\mu\nu}}{d\tau}} - \lambda_2 \tau_2^{(2)} \frac{d^2 E^{\mu\nu}}{d\tau^2} - \cdots .$$

Leading order, Newtonian response

Interpret $\tau_2^{(1)}$ as tidal lag time (Darwin 1879)

$$Q^{\mu\nu} \approx -\lambda_2 E^{\mu\nu}\left(t - \tau_2^{(1)}\right) \approx -\lambda_2 E^{\mu\nu} - \lambda_2 \tau_2^{(1)} \frac{dE^{\mu\nu}}{d\tau}.$$



Tidal deformabilities 1

- Stellar compactness

$$C_A \equiv \frac{m_A}{R_A}\,.$$

- Tidal deformability (Flanagan+Hinderer 2007)

$$\Lambda_A \equiv \frac{\lambda_{2,A}^{(0)}}{m_A^5} = \frac{2}{3}\frac{k_{2,A}}{C_A^5},$$

- Dissipative tidal deformability

$$\Xi_A \equiv -\frac{\lambda_{2,A}^{(0)}\tau_{2,A}^{(1)}}{m_A^6} = \frac{2}{3}\frac{k_{2,A}}{C_A^6}\frac{\tau_{d,A}}{t_R}.$$

Tidal deformabilities 2

- Tidal lag τ_d the new physical parameter to compute/constrain

$$\Xi_A \equiv -\frac{\lambda^{(0)}_{2,A}\tau^{(1)}_{2,A}}{m_A^6} = \frac{2}{3}\frac{k_{2,A}}{C_A^6}\frac{\boxed{\tau_{d,A}}}{t_R}.$$

- Many different physical processes can contribute to τ_d
  - Molecular viscosity of the star
  - Turbulent effective viscosity (Zahn 1966)
  - Dynamics of the stars crust (Kochanek 1992)
  - Out-of-equilibrium Urca processes (Haensel 2002, Alford+ 2018, Most+ 2021)

Tidal deformabilities 3

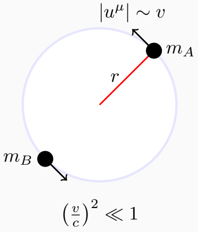
- Tidal response in frequency space

$$Q_{\mu\nu}(\omega) = F_2(\omega) E_{\mu\nu}(\omega) = (\Lambda(\omega) + \Xi(\omega)) E_{\mu\nu}(\omega).$$

- Dynamical tide $\Lambda(\omega)$
  - Conservative: $\Lambda(-\omega) = \Lambda(\omega)$
  - Captures excitation of normal modes of the star
- Dissipative tidal deformability Ξ
  - Higher order expansion $\Xi(\omega)$ is odd: $\Xi(-\omega) = -\Xi(\omega)$
- Damped simple harmonic oscillator (c.f. Sridhar+Tremaine 1992)

$$\begin{aligned} Q_{\mu\nu}(\omega) &= \frac{A}{\omega^2 - \omega_0^2 - i\gamma\omega} E_{\mu\nu}(\omega) \\ &= -\frac{A}{\omega_0^2}\left(\left(\frac{\omega}{\omega_0}\right)^2 - i\frac{\gamma\omega}{\omega_0^2} + \cdots\right) E_{\mu\nu}(\omega). \end{aligned}$$

Newtonian dynamics of a binary 1



- Newtonian equations of motion

$$a_i = \frac{GM}{r}\partial_i \frac{1}{r} + \frac{GM}{2}\left(\frac{Q_A^{<jk>}}{m_A} + \frac{Q_B^{<jk>}}{m_B}\right)\partial_i\partial_j\partial_k \frac{1}{r}.$$

- Newtonian tidal response

$$Q_A^{ij} = m_A^5\left(\Lambda_A E_A^{ij} - m_A \Xi_A \frac{dE_A^{ij}}{dt}\right).$$

Newtonian dynamics of a binary 2

Change in orbital energy

$$\frac{dE_{orb}}{dt} = \mathcal{F}_{diss}.$$

where

$$\begin{aligned} E_{orb} =& \frac{1}{2}\mu v_i v^i - \frac{\mu M}{r} - \frac{3\mu M}{2r^6}\left(m_B m_A^4 \Lambda_A + m_A m_B^4 \Lambda_B\right), \\ \mathcal{F}_{diss} =& -\frac{9\mu M}{r^8}\left(m_B m_A^5 \Xi_A + m_A m_B^5 \Xi_B\right)\left(2\dot{r}^2 + v_i v^i\right) . \end{aligned}$$

Gravitational wave phase

- Phasing formula

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

- Energy

$$E_{tot} = E_{orb}, \quad \frac{dE_{tot}}{dt} = \mathcal{F}_{diss} + \mathcal{F}_{GW}.$$

- Integrate twice, obtain **4PN** dissipative correction

$$\Psi(f) = \frac{3}{128}\frac{1}{\eta}u^{-5}\left[1 - \frac{75}{4}\bar{\Xi}\, u^8 \log(u) - \frac{39}{2}\bar{\Lambda}u^{10}\right] + 2\pi f \bar{t}_c - \varphi_c - \frac{\pi}{4},$$
$$u \equiv (Mf)^{1/3}.$$

Effacement principle

$$Q_{ij} = -\Lambda E_{ij} - \Xi \partial_t E_{ij}.$$

- **Effacement principle:** finite size of star affects equations of motion at **5PN** order (Damour 1987).
    - Dissipative finite size of star effects enter equations of motion at **6.5PN** order
- Q: How do dissipative, finite size effects enter at **4PN** in the GW phase?
- A: Dissipation affects $\dot{E}_{tot}$ through $\mathcal{F}_{diss}$

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

Assumptions

- No initial spin
- Negligible tidal spinup of stars
    - This approximation breaks down (stars become tidally locked) for white dwarfs (Burkart+ 2013)
- Ignore heating/finite temperature effects
- Orbital frequency far away from any stellar resonances
    - Could break down in the presence of low-frequency, highly stratified (g-) modes

Self-consistency of Newtonian calculation: no-spin calculation

- Tidal torquing spins-up stars

$$\frac{d\Omega_A}{dt} \approx \frac{45 G m_B^2 m_A^5}{2 R_A^2 M^6 c} \Xi_A \gamma_0^6 \left(\omega_A - \Omega_A\right).$$

- Inspiral driven by gravitational radiation reaction

$$\frac{dr}{dt} \approx -\frac{64 \eta c^3}{GM} \gamma_0^4.$$

- $\nu \gtrsim$ average causal bound in order for appreciable spinup of stars (tidal locking) before merger (Bildsten+Cutler 1992)

$$\frac{T_{lock}}{T_{insp}} \approx 0.4 \left(\frac{M}{3.2 M_\odot}\right)^3 \left(\frac{m_A}{1.6 M_\odot}\right)^3 \left(\frac{1.6 M_\odot}{m_B}\right) \left(\frac{12 km}{R_A}\right)^5 \left(\frac{\nu_c}{\nu}\right) \times \left(\frac{10}{p_{2,A}}\right) \left(\frac{0.1}{k_{2,A}}\right)$$

Measurability of the tidal dissipation

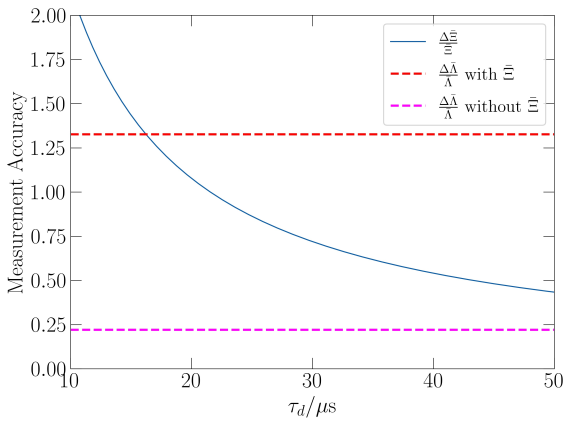
Fisher analysis for a quasi-circular binary 1

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\Psi(f)}.$$

$$\rho = 4 \int_{f_{lower}}^{f_{upper}} df \, \frac{\left|\tilde{h}(f)\right|^2}{S_n(f)}.$$

- $\Psi = \Psi_{pp} + \Psi_{\Lambda} + \Psi_{\Xi}$
- Ψ_{pp}: point particle, 4 PN (Blanchet+ 2023)
- Ψ_{Λ}: tidal Love, 5 PN (Flanagan+Hinderer 2008)
- Ψ_{Ξ}: dissipative, 4 PN (JLR+ 2023)
- GW170817-like parameters for O5 with SNR $= 100$.

Fisher analysis for a quasi-circular binary 2



$$\Xi_A = \frac{2}{3}\frac{k_{2,A}}{C_A^6}\frac{\tau_{d,A}}{t_{R,A}}.$$

Can place constraint if $\tau_d 20\mu s$

Mapping to microphysics

- Connect tidal delay to effective viscosity

$$\tau_{d,A} = \frac{p_{2,A} \nu_A R_A}{m_A}.$$

- $\nu_A = \langle \eta \rangle / \rho$: effective average kinematic viscosity (could include bulk viscosity)
- $p_{2,A}$: dimensionless coefficient to compute
- Many different physical processes could contribute to ν_A

Calculating the tidal response: black holes

$$Q_{\mu\nu}(\omega) = F_2(\omega) E_{\mu\nu}(\omega),$$

- Tidal response of black holes (Poisson 2009)

$$F_2(\omega) \sim 10^{-6} s \left(\frac{M}{20 M_\odot}\right) i\omega + \mathcal{O}\left(\omega^2\right).$$

Calculating the tidal response: relativistic stars

- Newtonian stars: Ξ computed for special cases
- Work in progress: general perturbative calculation

Calculating the tidal response: Newtonian stars 1

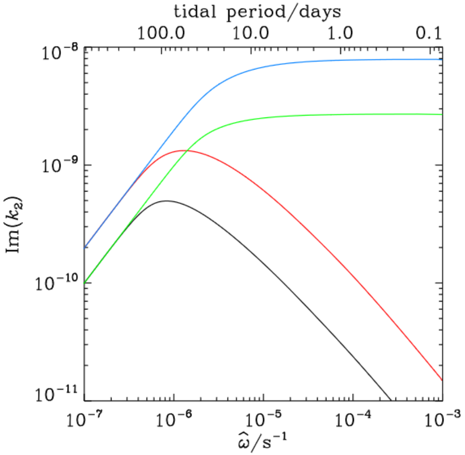
- Frequency space response

$$Q_{ij}(\omega) = F_2(\omega)\, E_{ij}(\omega)\,.$$

- $F_2(\omega)$: large number of calculations for main-sequence stars (e.g. Ogilvie 2014)
    - Turbulent dissipation main contributor to ν (Zahn 1976)

Calculating the tidal response: Newtonian stars 2

Response is complicated for main-sequence stars (Ogilvie 2014)



Calculating the tidal response: Newtonian stars 3

- Tidal response of white dwarfs (Campbell 1984, Willems+ 2010)
- Tidal response of neutron stars (Lai 1994)
- Revisit, consider simple case (polytropic star)

How important could bulk viscosity be? Revisiting the Newtonian case

- Navier-Stokes equations

$$
\begin{aligned}
D\rho + \rho\theta &= 0, \\
\rho D v^i + \nabla^i p - \nabla_k \left(\zeta\theta g^{jk} + 2\eta\sigma^{jk}\right) &= \rho\nabla^j U + \rho\nabla^j V, \\
\nabla_i \nabla^i U + 4\pi G\rho &= 0.
\end{aligned}
$$

- ζ: Bulk viscosity, θ: fluid expansion
- η: Shear viscosity, σ_{ij}: fluid shear
- V: External tidal field

Formal solution to Newtonian problem 1

- Linearly perturb Navier-Stokes equations, expand in *modes*
- Adiabatic, Lagrangian perturbation $\delta v^i = \partial_t \xi^i \to -i\omega\tilde{\xi}^i$
- Perturbed Navier-Stokes equations

$$\mathfrak{L}^i_j \tilde{\xi}^j + \delta\mathfrak{L}^i_j \tilde{\xi}^j - \omega^2 \tilde{\xi}^i = f^i.$$

- $\mathfrak{L}^i_j$: self-adjoint perfect-fluid differential operator (Chandrasekhar 1961)
- $\delta\mathfrak{L}^i_j$: viscous perturbation
- f^i: tidal forcing

Formal solution to Newtonian problem 2

$$Q_{ij}(\omega) = F_2(\omega)\, E_{ij}(\omega)\,.$$

- $\delta\mathfrak{L}^i_j = 0$: **assume mode solutions complete**
  - f, p, g modes
- Set $\delta\mathfrak{L}^i_j \neq 0$, compute perturbation to ω

$$\begin{aligned}
F_2 = {} & \frac{1}{2(2\ell+1)} \sum_{kn} \frac{\left(\bar{\nabla}\bar{r}^\ell, \bar{\xi}_k\right) \delta_{kn} \left(\bar{\nabla}\bar{r}^\ell, \bar{\xi}_n\right)}{\left(\omega_n^2 - \omega^2\right)\left(\bar{\xi}_n, \bar{\xi}_n\right)} \\
& - \frac{i\omega\bar{\nu}}{2(2\ell+1)\nu_c} \sum_{kn} \frac{\left(\bar{\nabla}\bar{r}^\ell, \bar{\xi}_k\right) \delta\mathfrak{L}_{kn} \left(\bar{\nabla}\bar{r}^\ell, \bar{\xi}_n\right)}{\left(\omega_n^2 - \omega^2\right)\left(\bar{\xi}_n, \bar{\xi}_n\right)} \\
& + \mathcal{O}\left[\left(\frac{\bar{\nu}}{\nu_c}\delta\mathfrak{L}_{kn}\right)^2\right]\,.
\end{aligned}$$

Formal solution to Newtonian problem: detectability

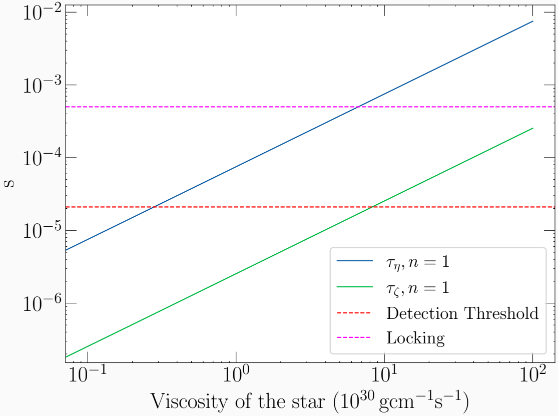
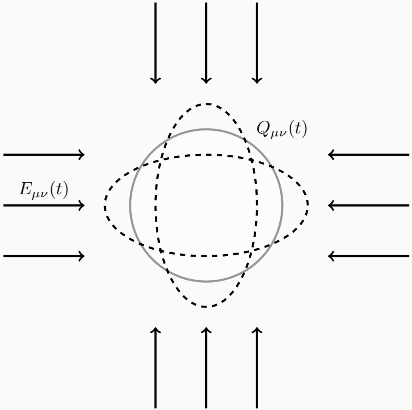


Figure 8: Dissipation time scale set by viscosity

- $\eta \lesssim 10^{29} \mathrm{g\ cm^{-1}\ s^{-1}}$ (Kochaneck 1992, Shternin+Yakovlev 2008)
- $\zeta \lesssim 10^{29} \mathrm{g\ cm^{-1}\ s^{-1}}$ (Most+ 2021)

Why is bulk viscosity less important? (Newtonian case)



- Dominant contribution to gravitational coupling (f-mode) induces very little compression in the star
- Future work: determine if there is more compression for relativistic stars

- No complete set of modes
- Spacetime is dynamical

$$g^{\mu\nu} = g^{\mu\nu}_{(0)} + \delta g^{\mu\nu}, \quad \cdots$$

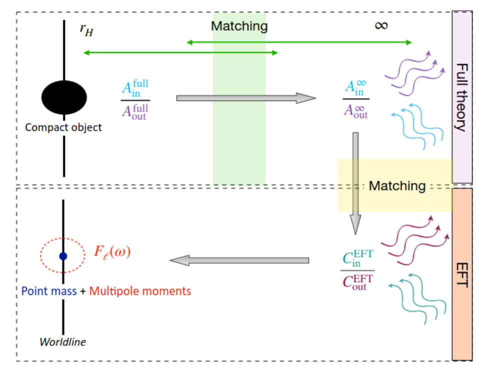


Figure 9: Creci, Hinderer, Steinhoff, 2108.03385

(Weak) constraints on out-of-equilibrium physics

$$h(f) = A(f)\, e^{i\Psi(f)}$$

$$\Psi \to \Lambda \to p(\rho)$$

$$\Psi \to \Xi \to \zeta(\rho)\,,\ \eta(\rho)$$

Conclusion

- Dissipative effects in neutrons star may be measurable during the inspiral of a binary
- Tidal deformability Λ + dissipative tidal deformability Ξ: two numbers to probe neutron star microphysics
- Complements constraints that could come from merger/post merger
- More details: **2306.15633**
- Bulk viscosity more difficult to constrain than shear viscosity
- Future work
  - Compute viscous contribution to Ξ
  - Compute leading PN corrections to phase
  - Relax assumptions: add initial NS spin
  - Obtain constraint on Ξ from GW170817

Backup slides

Modes of a star

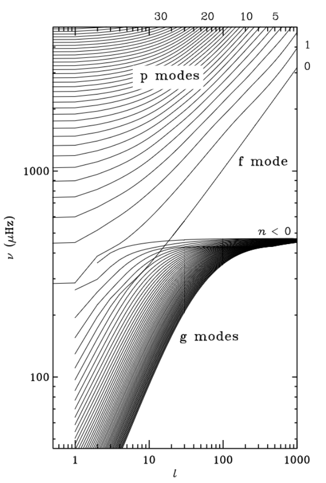


Figure 1: Christensen-Dalsgaard, Lecture notes on stellar oscillations, 1998

Bulk viscosity function of density

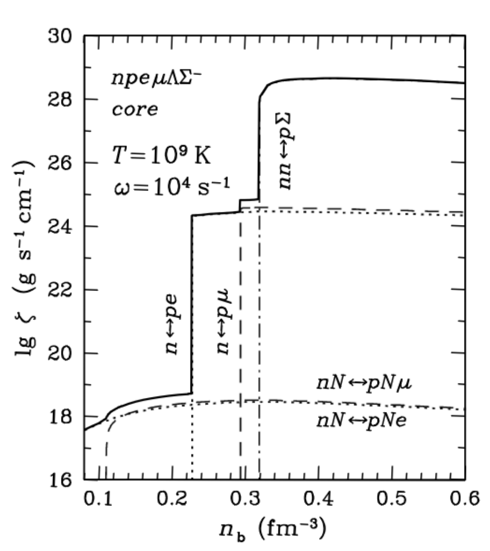


Figure 2: Haensel, Potokhin, Yakovlev, Neutron Stars 1, 2007

Adiabatic Love numbers

1. Spherically symmetric star (TOV equations)

$$g^{(0)}_{\mu\nu},\ \delta u^{\mu}_{(0)},\ \rho_{(0)},\ \dots\ \rightarrow\ G^{(0)}_{\mu\nu} = \frac{8\pi G}{c^4} T^{(0)}_{\mu\nu}.$$

2. **Adiabatic** (time independent) linear perturbation (Hinderer 2008, Damour+Nagar 2009, Binnington+Poisson 2009)

$$\delta g_{\mu\nu},\ \delta u^{\mu},\ \delta\rho,\ \dots\ \rightarrow\ \delta G_{\mu\nu} = \frac{8\pi G}{c^4}\delta T_{\mu\nu}.$$

3. Extract quadrupole from g_{tt} (Thorne 1998, Hinderer 2008)

No viscous corrections to stress-energy tensor

1. Perturbed stress energy reduces to perfect fluid.

$$\delta T_{\mu\nu} = \delta \left(\frac{e}{c^2} u_\mu u_\nu + p \Delta_{\mu\nu} \right).$$

2. There are no viscous corrections to the adiabatic tidal Love numbers.
3. Can extend argument to other fluid models.

Relativistic, viscous fluids

Relativistic, causal, hyperbolic theory of viscous fluids: **BDNK fluid** (Kovtun 2019, Bemfica+ 2020)

$$
\begin{aligned}
T_{\mu\nu} =& \mathcal{E} u_\mu u_\nu + \mathcal{P} \Delta_{\mu\nu} + \boxed{2\mathcal{Q}_{(\mu} u_{\nu)}} - \boxed{2\eta\, \sigma_{\mu\nu}}, \\
\mathcal{E} \equiv& \frac{1}{c^2} \left(e + \boxed{\tau_\epsilon \left[u^\alpha \nabla_\alpha e + (e+p)\,\theta \right]} \right), \\
\mathcal{P} \equiv& p - \boxed{\zeta\theta} + \boxed{\tau_p \left[u^\alpha \nabla_\alpha e + (e+p)\,\theta \right]}, \\
\mathcal{Q}_\mu =& \boxed{\tau_Q \left(\tfrac{(e+p)}{c^2} u^\alpha \nabla_\alpha u_\mu + \Delta^\alpha_\mu \nabla_\alpha p \right)} \\
& + \boxed{\frac{\rho\kappa T^2}{m_b (e+p) c^2} \Delta^\alpha_\mu \nabla_\alpha \left(\frac{\mu}{T} \right)}..
\end{aligned}
$$

More facts about the BDNK fluid model

- **BDNK fluid** (Kovtun 2019, Bemfica+ 2020) is the **only** relativistic fluid model that
  1. Is causal and strongly hyperbolic.
  2. Has stable equilibrium states.
  3. Includes bulk viscosity, shear viscosity, and heat conduction.
  4. Includes nonzero baryon number.
  5. Entropy increases with time.

More facts about the BDNK fluid model

1. Fluid current

$$J^\mu = \rho u^\mu.$$

2. Set $\tau_\epsilon = 0$, $\tau_p = 0$, $\tau_Q = 0$, and the theory reduces to an **Eckart fluid** (Eckart 1940).
3. Requiring that the theory by hyperbolic in the relativistic regime, and reduce to the Navier-Stokes equations of motion at 0PN constrains the heat conductivity and shear viscosity to satisfy $\kappa > \eta k_B/m_b$ (Hegade+ 2023).