Probing the internal dynamics of neutron stars with gravitational waves

ARC Seminar, UIUC

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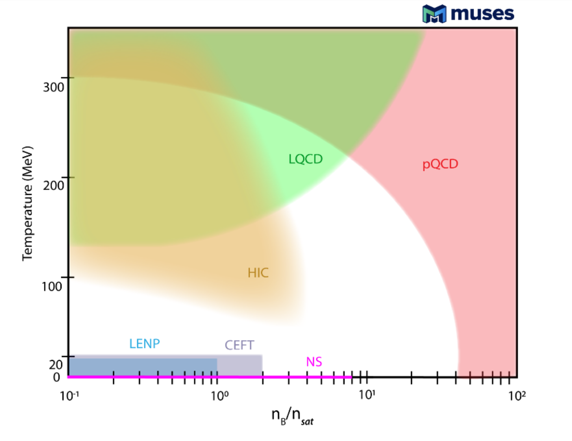
Abhishek Hegade, Rohit Chandramouli, Nicolás Yunes

University of Illinois Urbana-Champaign & ICASU
2306.15633, **2312.11659**; 24–.—

What are the properties of high-density nuclear matter?



$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi$$

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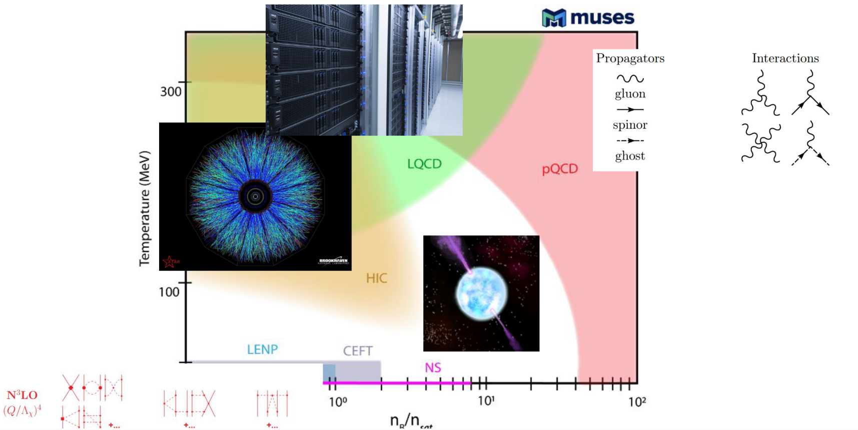

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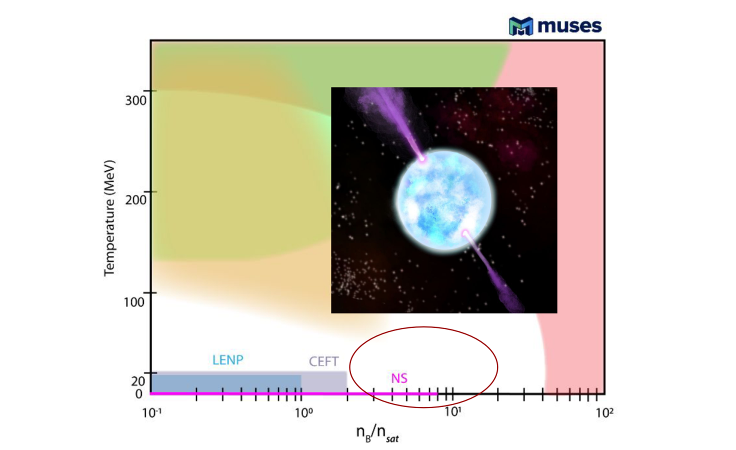

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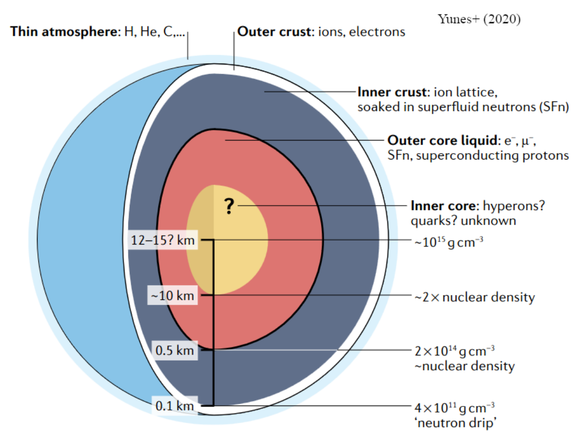
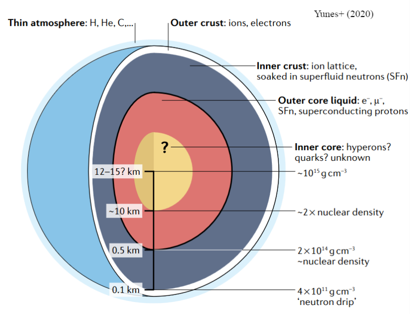


Yunes+ Nature Rev.Phys. 4 (2022) 4, 237-246

Neutron stars: relativistic, viscous, fluid

1. Density ρ, temperature T



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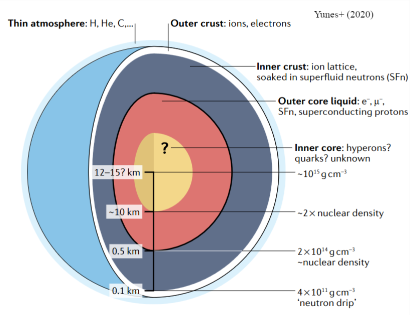

Neutron stars: relativistic, viscous, fluid

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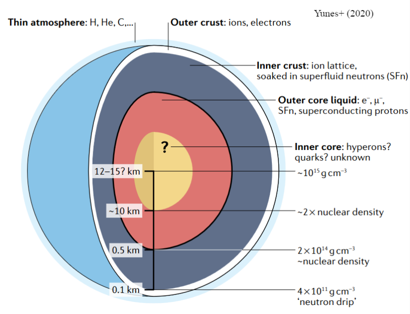

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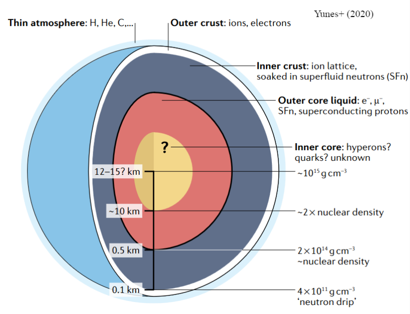

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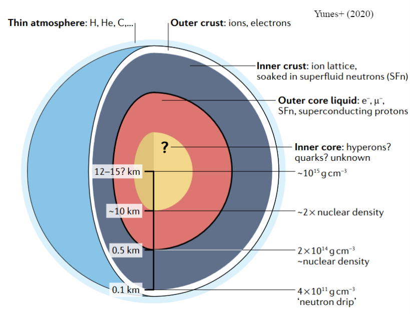


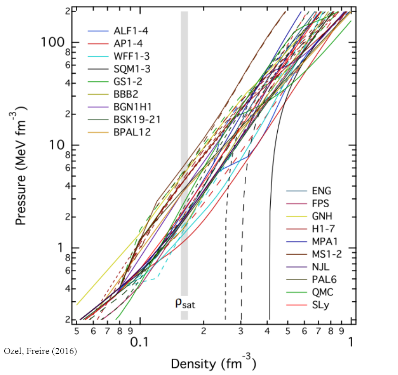
$$T_{\mu\nu} = (\rho + p + \zeta\theta)\, u_\mu u_\nu + p g_{\mu\nu} - 2\eta\sigma_{\mu\nu} + 2q_{(\mu} u_{\nu)} + \cdots .$$

Equation of state: $p(\rho)$

Strong Force → $p(\rho)$

Weak Force → $p(\rho)$

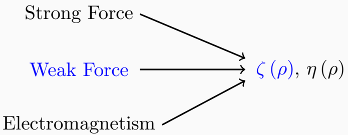
Electromagnetism → $p(\rho)$

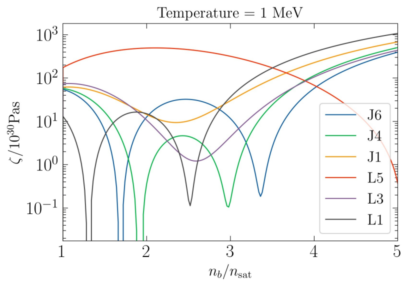


Ozel, Freire (2016)

Neutron star viscosity

Viscosity: $\{\zeta(\rho), \eta(\rho)\}$





Yang+ (2023); Hegade, Yang, Noronha, Teixera, Noronha-Hostler, Yunes, JLR, ... (work in progress...)

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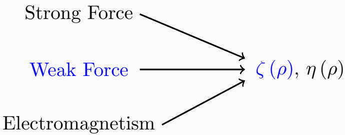
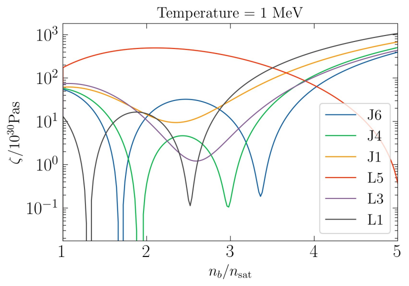


Neutron stars: $\eta \ll \zeta$ (Sawyer 1989)



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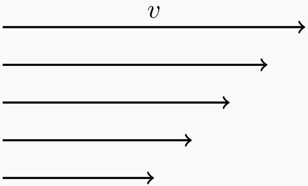

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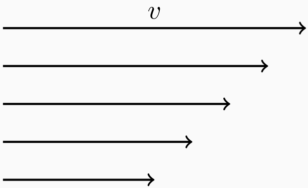

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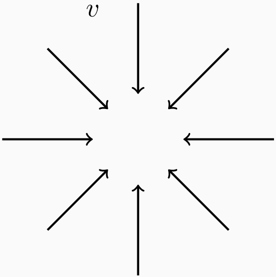
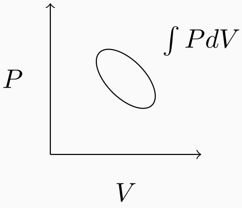


$$\Delta S \sim \zeta \theta^2 \sim \zeta \left(\Delta V\right)^2$$

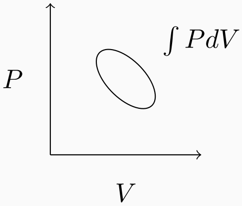
What is bulk viscosity?



$$\Delta E = T\Delta S - P\Delta V.$$

1. Phase lag in system response

What is bulk viscosity?



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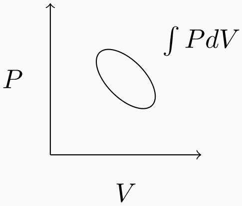

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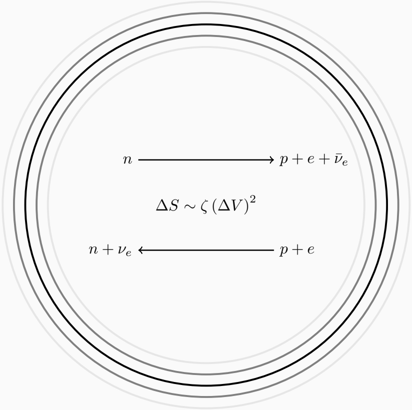
1. Phase lag in system response
2. Internal property of system (Urca processes: proton fraction) takes time to equilibrate
3. Out of phase: work: dissipation

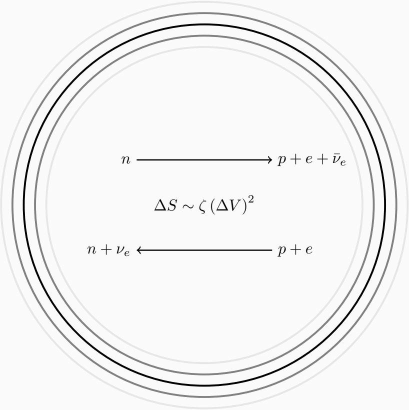
Bulk viscosity in neutron stars: Urca processes (Sawyer 1989)

1. Neutron star:
   $\zeta \sim 10^{31-32}\ \mathrm{g\ cm^{-1}s^{-1}}$ (Yang+ 2023)

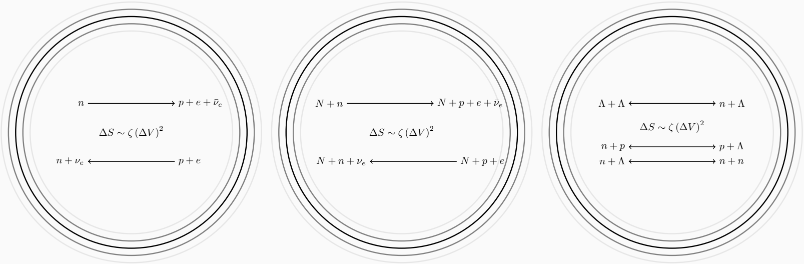


Bulk viscosity in neutron stars: Urca processes (Sawyer 1989)

1. Neutron star: $\zeta \sim 10^{31-32}\ \mathrm{g\ cm^{-1} s^{-1}}$ (Yang+ 2023)
2. Pitch: $\eta \sim 10^{9}\ \mathrm{g\ cm^{-1} s^{-1}}$



Particle conversion: temperature dependence



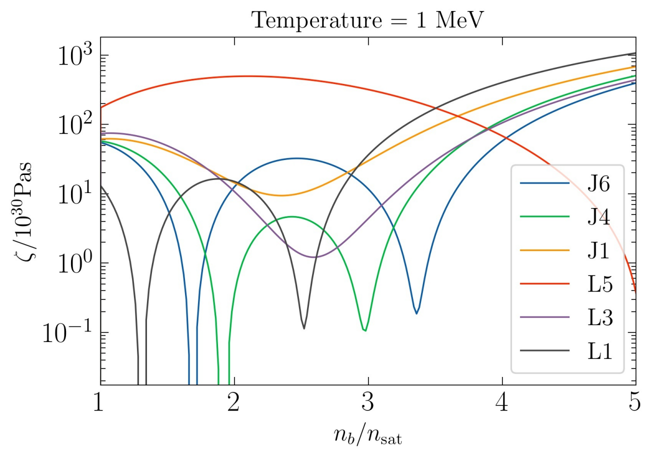
$$\zeta = \zeta_{Urca} + \zeta_{mUrca} + \zeta_{Hyperon} \sim 10^{26-32} \frac{\mathrm{g}}{\mathrm{cm\ s}}$$

$\zeta_{Urca} \propto T^4$ $\qquad$ $\zeta_{mUrca} \propto T^6$ $\qquad$ $\zeta_{Hyperon} \propto T^{-2}$

Sawyer (1989), Yakovlev+Levenfish (1994), Jones (2001), Linblom+Owen (2002), Most+ (2021), Yang+ (2023), ...

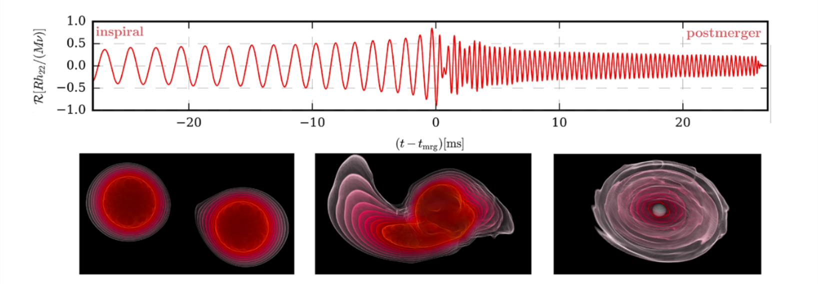
Different assumptions: different values at a fixed temperature

$\zeta(\rho)$



Yang+ (2023); **24–.**— Hegade, Yang, Teixera, Noronha, Noronha-Hostler, Yunes, JLR, ... (work in progress...)

Neutron star binaries



Dietrich+ (2021)

How well can we constrain $p(\rho)$ and $\zeta(\rho)$ with gravitational wave observations of the **inspiral** of neutron star binaries?

Previous work

- Viscous effects and tidal locking (Bildsten+Cutler 1992)

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- Specific PN contribution of bulk viscosity during inspiral not measurable (Most+ 2021)

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Projecting the likely importance of weak-interaction-driven bulk viscosity in neutron star mergers

Elias R. Most,[1,2,3★] Steven P. Harris,[4★] Christopher Plumberg,[5] Mark G. Alford,[6] Jorge Noronha,[5★] Jacquelyn Noronha-Hostler,[5] Frans Pretorius,[2,7] Helvi Witek[5] and Nicolás Yunes[5]

Main conclusions

$$Q_{ij} \sim -\Lambda E_{ij} - \Xi \partial_t E_{ij}.$$

- **Tidal deformability** Λ: correction to GW phase due to equilibrium properties of star $p(\rho)$

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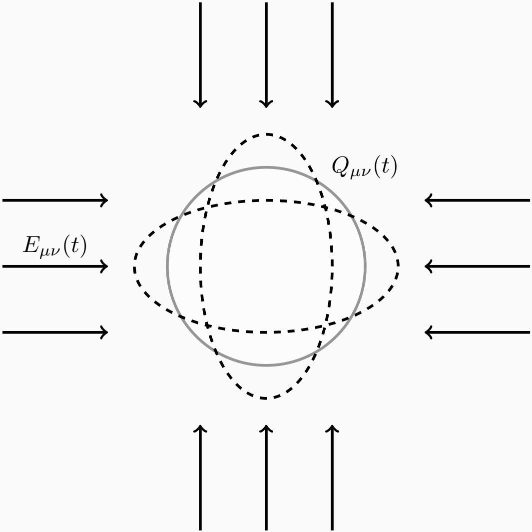
- **Dissipative tidal deformability** Ξ: correction to GW phase from out-of-equilibrium, dissipative properties of star $\zeta(\rho)$
- **Physically allowed values of viscosity may be constrained with inspiral GW data from ground-based detectors**

Overview of rest of talk

1. Tidal response of a relativistic star
2. Computing the GW phase
3. Measuring tides from GW170817 & implications for nuclear physics

Tidal response of a (relativistic) star



24–.–, Abhishek Hegade, JLR, Nicolás Yunes

Tidal response of a star

- Assume slowly changing tidal field

$$\frac{dE^{\mu\nu}}{d\tau} \ll \omega_f E_{\mu\nu}, \quad \frac{d^2 E^{\mu\nu}}{d\tau^2} \ll \omega_f \frac{dE^{\mu\nu}}{d\tau}, \cdots$$

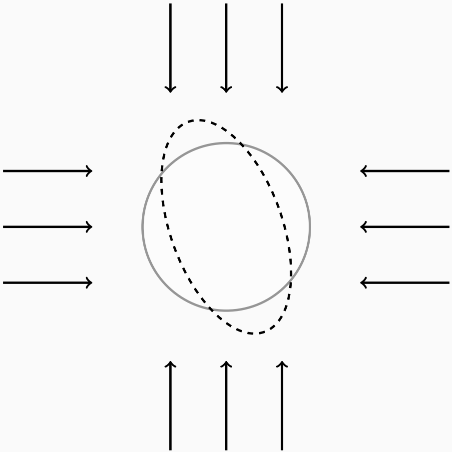
- Assume linear quadrupolar response

$$Q^{\mu\nu} = \boxed{-\lambda_2 E^{\mu\nu} - \lambda_2 \tau_2^{(1)} \frac{dE^{\mu\nu}}{d\tau}} - \lambda_2 \tau_2^{(2)} \frac{d^2 E^{\mu\nu}}{d\tau^2} - \cdots .$$

Leading order, Newtonian response

Interpret $\tau_2^{(1)}$ as tidal lag time (Darwin 1879)

$$Q^{\mu\nu} \approx -\lambda_2 E^{\mu\nu}\left(t - \tau_2^{(1)}\right) \approx -\lambda_2 E^{\mu\nu} - \lambda_2 \tau_2^{(1)} \frac{dE^{\mu\nu}}{d\tau}.$$



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$$Q_{\mu\nu} \approx -m_A^5\Lambda_A E_{\mu\nu} + m_A^6\Xi_A\partial_t E_{\mu\nu}.$$

Tidal deformabilities 2

- Tidal response in frequency space

$$Q_{\mu\nu}(\omega) = F_2(\omega)\, E_{\mu\nu}(\omega) = \left(\Lambda(\omega) + \Xi(\omega)\right) E_{\mu\nu}(\omega)\,.$$

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- Damped simple harmonic oscillator

$$\begin{aligned} Q_{\mu\nu}(\omega) &= \frac{A}{\omega^2 - \omega_0^2 - i\gamma\omega} E_{\mu\nu}(\omega) \\ &= -\frac{A}{\omega_0^2}\left(1 - i\frac{\gamma\omega}{\omega_0^2} + \mathcal{O}\left(\omega^2\right)\right) E_{\mu\nu}(\omega) \,. \end{aligned}$$

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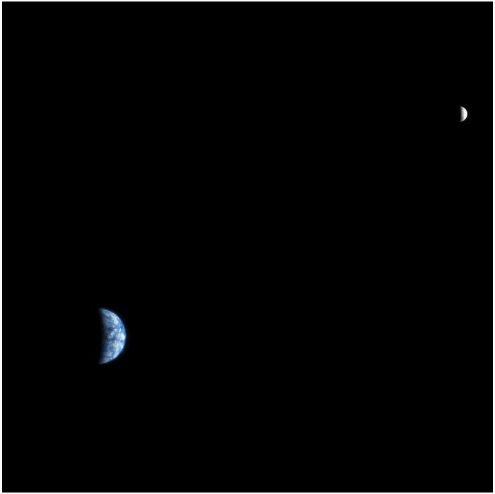
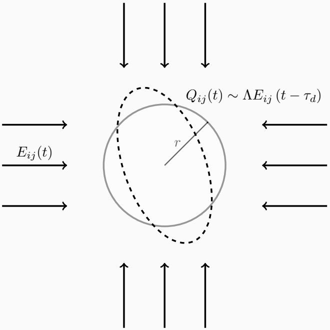
- Contribution to $\tau_{d,A}, \Xi_A$: $\zeta(\rho)$, $\eta(\rho)$ (Hegade+JLR+Yunes, in prep)

Tides in our solar system



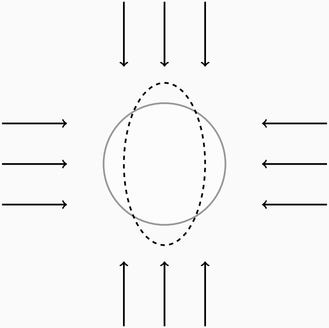
NASA

The moon and tides



NASA

Tides and relativistic stars



Perturbation theory about a static star

$$
\begin{aligned}
\delta G_{\mu\nu} &= 8\pi \delta T_{\mu\nu} \\
\delta\left(\nabla_\mu T^{\mu\nu}\right) &= 0 \\
T_{\mu\nu} &= \left(\rho + p + \zeta\theta\right) u_\mu u_\nu + p g_{\mu\nu} - 2\eta\sigma_{\mu\nu} + \cdots \\
p\left(\rho\right), \quad \zeta\left(\rho\right) \quad \ldots
\end{aligned}
$$

Brief overview of stellar perturbation theory

(Thorne + Campolattaro 1967)

1. Metric perturbation

$$\begin{aligned} g_{\mu\nu} dx^\mu dx^\nu = & -e^{\nu(r)} \left(1 - 2H(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) dt^2 \\ & - 2iH_1(r) e^{-i\omega t} r^\ell Y_{\ell m} dt dr \\ & + e^{\lambda(r)} \left(1 + 2H_2(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) dr^2 \\ & + r^2 \left(1 - K(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) d\Omega^2 \end{aligned}$$

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$$\begin{aligned} g_{\mu\nu} dx^\mu dx^\nu = &-e^{\nu(r)} \left(1 - 2H(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) dt^2 \\ &- 2iH_1(r) e^{-i\omega t} r^\ell Y_{\ell m} dt dr \\ &+ e^{\lambda(r)} \left(1 + 2H_2(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) dr^2 \\ &+ r^2 \left(1 - K(r) e^{-i\omega t} r^\ell Y_{\ell m}\right) d\Omega^2 \end{aligned}$$

2. Fluid perturbation: Lagrangian perturbation theory: ξ^μ, $\Delta \equiv \delta + \mathcal{L}_\xi$

$$\begin{aligned} \delta u^t &= e^{-\frac{\nu}{2}} r^\ell H e^{-i\omega t} Y_{\ell m} \,, \\ \delta u^r &= -i\omega e^{-(\lambda+\nu)/2} r^{\ell-1} W e^{-i\omega t} Y_{\ell m} \,, \\ \delta u_A &= i\omega e^{-\nu/2} r^\ell V e^{-i\omega t} E_A^{\ell m} \\ \frac{\Delta p}{p} &= \gamma \frac{\Delta n}{n} \\ \Delta n &= \cdots . \end{aligned}$$

Brief overview of stellar perturbation theory

1. Can reduce to four first-order ordinary differential equations

$$\frac{d\vec{Y}}{dr} = \mathbf{A}\vec{Y} + \mathbf{B}\vec{S}\,,$$
$$\vec{Y} \equiv (H, W, V, H')\ ,$$
$$\vec{S} \equiv (S_0, S_1, S_Z, S_\Omega, S_0', S_1', S_Z', S_\Omega', S_1'')\ ,$$

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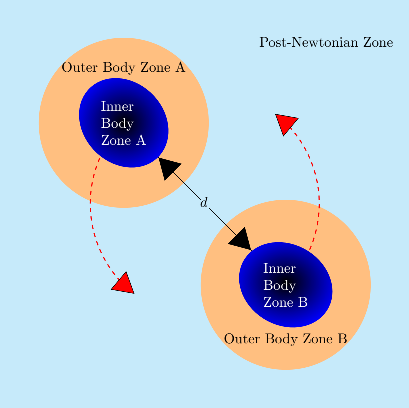
2. $\vec{Y}$: metric + perfect fluid terms (Linblom+Detweiler (1983), Detweiler+Linblom (1985), Lindblom+ (1997))
3. $\vec{S}$: viscous terms (**Hegade**+ (in prep))

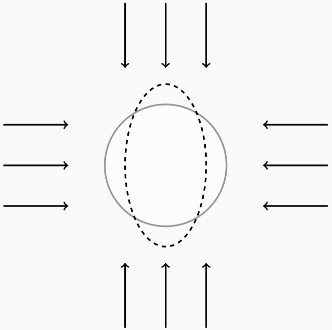
Matching to zones



Matching neutron star perturbation to external (Post-Newtonian) metric (Poisson 2020, Hegade + (in prep))

Stationary perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}(\mathbf{x})$$
$$P = P^{(0)} + \delta P(\mathbf{x})$$
$$\dots$$

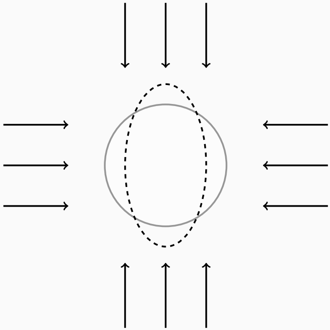


Λ_A and relativistic stars

Stationary perturbation

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}(\mathbf{x})$$
$$P = P^{(0)} + \delta P(\mathbf{x})$$
$$\ldots$$

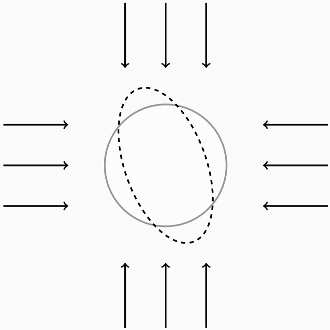
Λ: contribution from $p(\rho)$, no contribution from ζ, η (Flanagan+Hinderer (2007); Hinderer (2007), JLR, Hegade, Yunes (2023))



Ξ_A and relativistic stars

Slowly changing perturbation

$$\delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu}$$
$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}(t, \mathbf{x})$$
$$P = P^{(0)} + \delta P(t, \mathbf{x})$$
$$\dots$$
$$m_A \partial_t g_{\mu\nu} \ll 1$$
$$m_A \partial_t P \ll 1$$



Ξ_A and relativistic stars

Slowly changing perturbation

$$
\begin{aligned}
\delta G_{\mu\nu} &= 8\pi \delta T_{\mu\nu} \\
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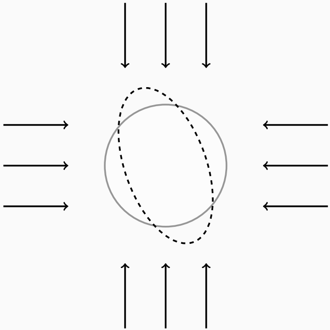
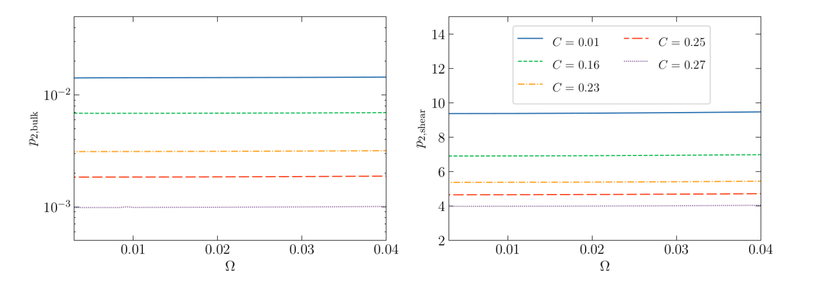
$$



$\Xi \propto \{\zeta, \eta\}$
(Hegade, JLR, Yunes (in prep))

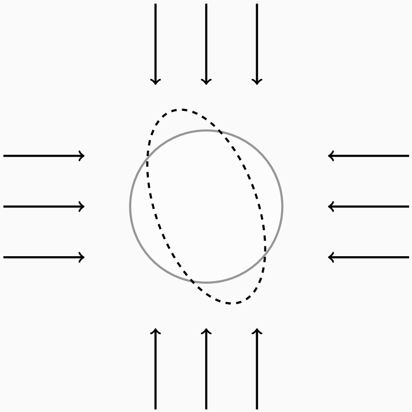
Ξ_A and relativistic stars



(Hegade, JLR, Yunes (in prep))

More shear than compression of star.

Ξ_A and relativistic stars



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Tides and black holes

1. Tidal deformability is zero (Binnington+Poisson 2009, Chia 2020)

$$\Lambda_A = 0$$

Tides and black holes

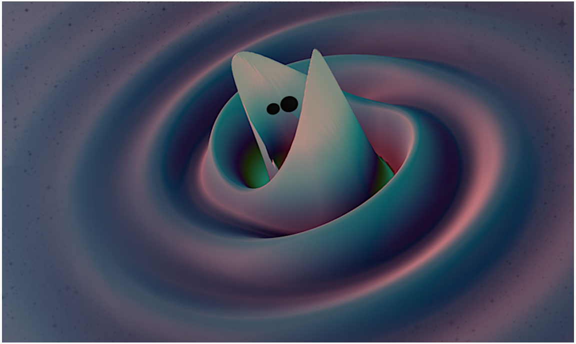
1. Tidal deformability is zero (Binnington+Poisson 2009, Chia 2020)

$$\Lambda_A = 0$$

2. Dissipative deformability is nonzero (Poisson 2009)

$$\Xi_A \sim 10^{-6} s \left(\frac{M}{20 M_\odot} \right).$$

Generation of gravitational waves from neutron star binaries

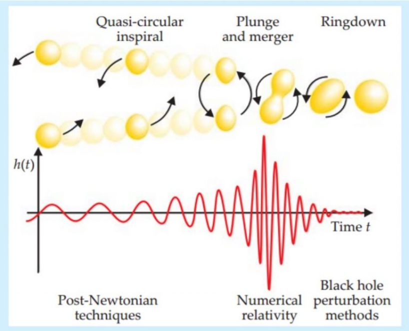


2306.15633, JLR, Abhishek Hegade, Nicolás Yunes

Generation of gravitational waves

Accelerating masses generate gravitational waves

$$\delta g_{\mu\nu} \equiv h_{\mu\nu}.$$



Baumgarte+Shapiro 2010

$$h_{ij}(t) \sim \ddot{I}_{ij}(t - r)$$

Computing the GW phase

- GW strain h

$$h(f) = A(f)\, e^{i\Psi(f)}.$$

- Differential equation for phase in terms of total binding energy E_{tot} of a binary (Tichy+ 2000)

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

- Compute $E_{tot}, \dot{E}_{tot}$ for a Newtonian binary, including tidal responses of each star

Gravitational waves and neutron star binaries

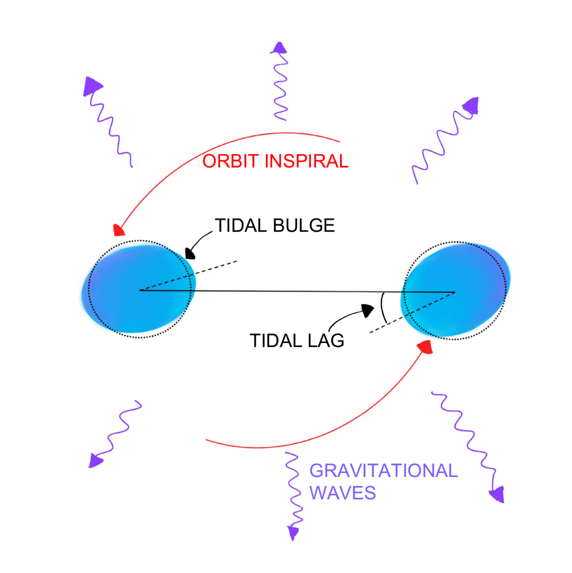
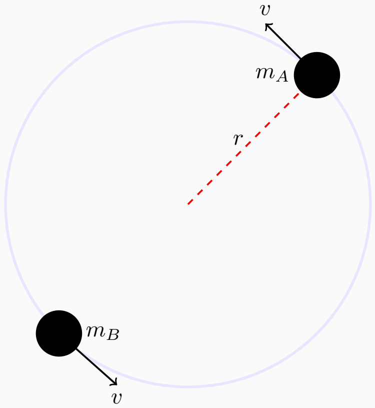


Image courtesy of Rohit Chandramouli

Newtonian dynamics of two point particles

Center of mass acceleration

$$a_i = \frac{M}{r}\partial_i \frac{1}{r}$$



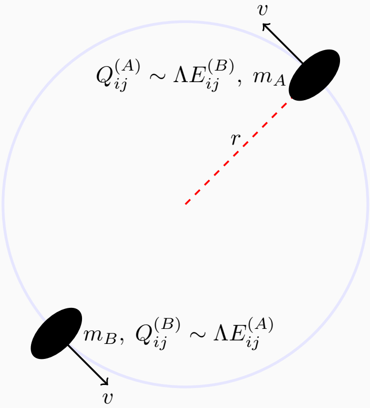
Newtonian dynamics with finite size corrections

Center of mass acceleration

$$a_i = \frac{M}{r}\partial_i \frac{1}{r} + \frac{M}{2}\left(\frac{Q_A^{<jk>}}{m_A} + \frac{Q_B^{<jk>}}{m_B}\right)\partial_i\partial_j\partial_k \frac{1}{r}.$$

Quadrupolar moment

$$Q_A^{ij} = m_A^5 \Lambda_A E_A^{ij}.$$



Newtonian dynamics with finite size + time corrections

Center of mass acceleration

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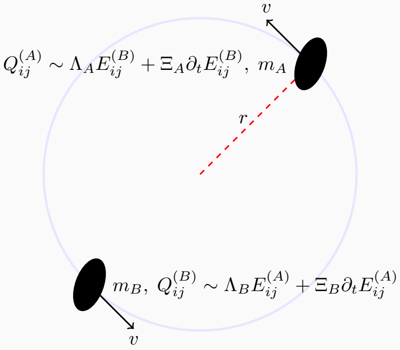
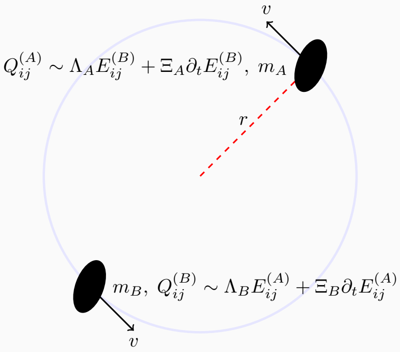
Quadrupolar moment

$$Q_A^{ij} = m_A^5 \left(\Lambda_A E_A^{ij} - m_A \Xi_A \frac{dE_A^{ij}}{dt}\right).$$



Energy equation



$$\frac{dE_{orb}}{dt} = \mathcal{F}_{diss}.$$

where

$$E_{orb} = \frac{1}{2}\mu v_i v^i - \frac{\mu M}{r} - \frac{3\mu M}{2r^6}\left(m_B m_A^4 \Lambda_A + m_A m_B^4 \Lambda_B\right),$$

$$\mathcal{F}_{diss} = -\frac{9\mu M}{r^8}\left(m_B m_A^5 \Xi_A + m_A m_B^5 \Xi_B\right)\left(2\dot{r}^2 + v_i v^i\right).$$

Gravitational wave phase

Phasing formula

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

Gravitational wave phase

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Gravitational wave phase

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Integrate twice, obtain

$$\Psi(f) = \frac{3}{128} \frac{1}{\eta_{SM}} u^{-5} \left[1 - \frac{75}{4} \bar{\Xi}\, u^8 \log(u) - \frac{39}{2} \bar{\Lambda} u^{10}\right] + 2\pi f t_c - \varphi_c - \frac{\pi}{4},$$

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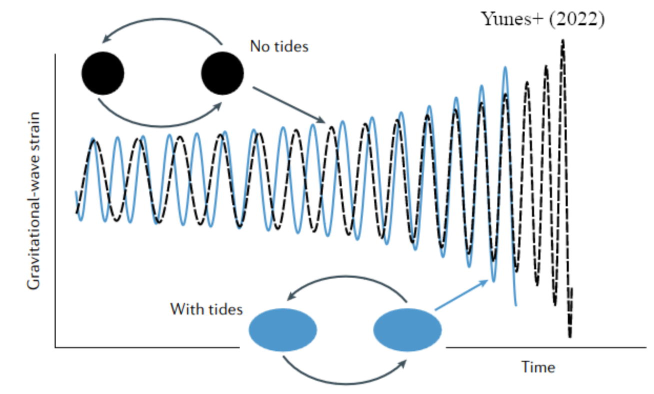
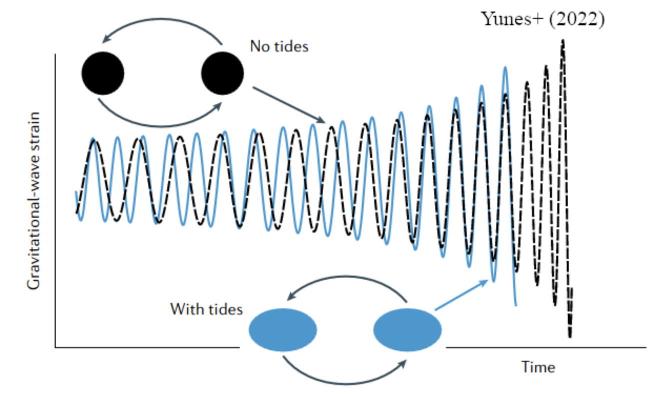
$$\begin{aligned}\bar{\Lambda} =& f_1(\eta_{SM})\frac{\Lambda_A + \Lambda_B}{2} + g_1(\eta_{SM})\frac{\Lambda_A - \Lambda_B}{2},\\ \bar{\Xi} =& f_2(\eta_{SM})\frac{\Xi_A + \Xi_B}{2} + g_2(\eta_{SM})\frac{\Xi_A - \Xi_B}{2}.\end{aligned}$$

Tidal contribution to gravitational waveform



$$\Delta\Psi_{\Lambda} = -\frac{117}{256}\frac{1}{\eta_{SM}}\bar{\Lambda}u^{5}, \qquad \Delta\Psi_{\Xi} = -\frac{225}{4096}\frac{1}{\eta_{SM}}\bar{\Xi}u^{3}\log\left(u\right).$$

Tidal contribution to gravitational waveform



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$\bar{\Xi}$ enters at lower PN order than $\bar{\Lambda}$

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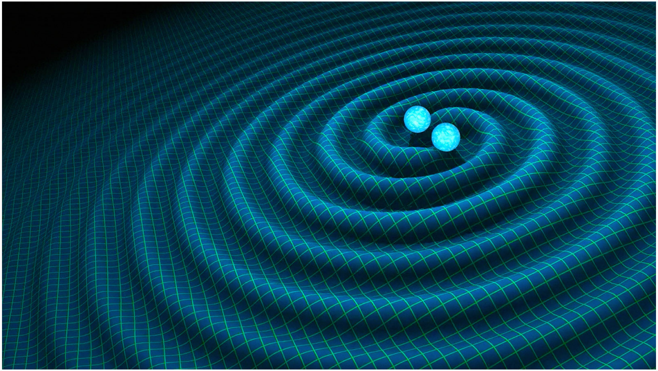
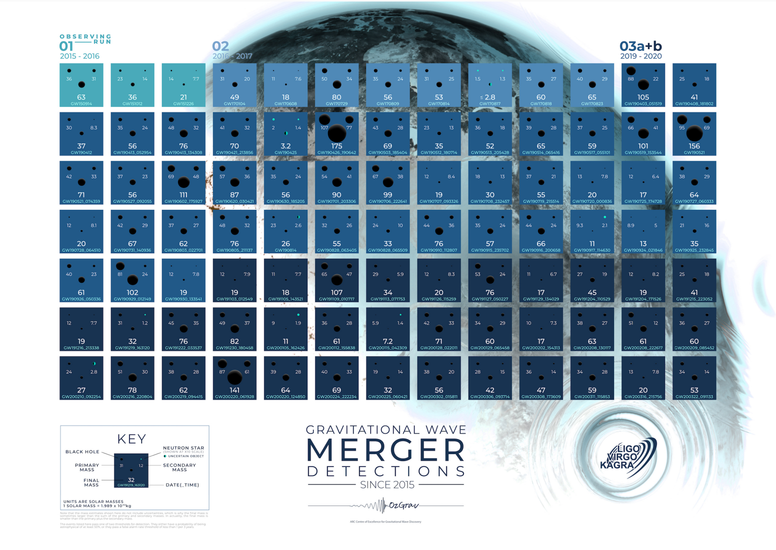
- Ignore heating/finite temperature effects
- Orbital frequency far away from any stellar resonances
  - Could break down in the presence of low-frequency, highly stratified (g-) modes

GW170817: what can the data tells us about NS tides?

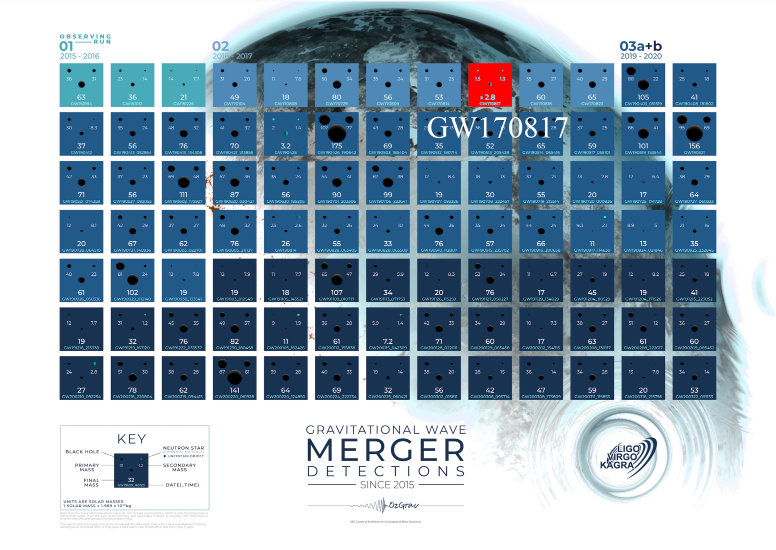


2312.11659: JLR, Abhishek Hegade, Rohit Chandramouli, Nicolás Yunes

GW170817


OBSERVING RUN
O1
2015 - 2016
O2
2016 - 2017
O3a+b
2019 - 2020
KEY
BLACK HOLE
NEUTRON STAR
PRIMARY MASS
SECONDARY MASS
FINAL MASS
DATE(_TIME)
UNITS ARE SOLAR MASSES
GRAVITATIONAL WAVE
MERGER
DETECTIONS
SINCE 2015
OzGrav
LIGO
VIRGO
KAGRA

GW170817


OBSERVING RUN
01
2015 - 2016
02
2016 - 2017
O3a+b
2019 - 2020
36 31 63 GW150914
23 14 36 GW151012
14 7.7 21 GW151226
31 20 49 GW170104
11 7.6 18 GW170608
50 34 80 GW170729
35 24 56 GW170809
31 25 53 GW170814
1.5 1.3 ≤2.8 GW170817
35 27 60 GW170818
40 29 65 GW170823
88 22 105 GW190403_051519
25 18 41 GW190408_181802
30 8.3 37 GW190412
35 24 56 GW190413_052954
48 32 76 GW190413_134308
41 32 70 GW190421_213856
2 1.4 3.2 GW190425
107 77 175 GW190426_190642
43 28 69 GW190503_185404
35 GW190512_180714
36 52 GW190513_205428
28 65 GW190514_065416
37 25 59 GW190517_055101
66 41 101 GW190519_153544
95 69 156 GW190521
GW170817
42 33 71 GW190521_074359
37 23 56 GW190527_092055
69 48 111 GW190602_175927
57 36 87 GW190620_030421
35 24 56 GW190630_185205
54 41 90 GW190701_203306
67 38 99 GW190706_222641
12 8.4 19 GW190707_093326
18 13 30 GW190708_232457
37 21 55 GW190719_215514
13 7.8 20 GW190720_000836
12 6.4 17 GW190725_174728
38 29 64 GW190727_060333
12 8.1 20 GW190728_064510
42 29 67 GW190731_140936
37 27 62 GW190803_022701
48 32 76 GW190805_211137
23 2.6 26 GW190814
32 26 55 GW190828_063405
24 10 33 GW190828_065509
44 36 76 GW190910_112807
35 24 57 GW190915_235702
44 24 66 GW190916_200658
9.3 2.1 11 GW190917_114630
8.9 5 13 GW190924_021846
21 16 35 GW190925_232845
40 23 61 GW190926_050336
81 24 102 GW190929_012149
12 7.8 19 GW190930_133541
12 7.9 19 GW191103_012549
11 7.7 18 GW191105_143521
65 47 107 GW191109_010717
29 5.9 34 GW191113_071753
12 8.3 20 GW191126_115259
53 24 76 GW191127_050227
11 6.7 17 GW191129_134029
27 19 45 GW191204_110529
12 8.2 19 GW191204_171526
25 18 41 GW191215_223052
12 7.7 19 GW191216_213338
31 1.2 32 GW191219_163120
45 35 76 GW191222_033537
49 37 82 GW191230_180458
9 1.9 11 GW200105_162426
36 28 61 GW200112_155838
5.9 1.4 7.2 GW200115_042309
42 33 71 GW200128_022011
34 29 60 GW200129_065458
10 7.3 17 GW200202_154313
38 27 63 GW200208_130117
51 12 61 GW200208_222617
36 27 60 GW200209_085452
24 2.8 27 GW200210_092254
51 30 78 GW200216_220804
38 28 62 GW200219_094415
87 61 141 GW200220_061928
39 28 64 GW200220_124850
40 33 69 GW200224_222234
19 14 32 GW200225_060421
38 20 56 GW200302_015811
28 15 42 GW200306_093714
36 14 47 GW200308_173609
34 28 59 GW200311_115853
13 7.8 20 GW200316_215756
34 14 53 GW200322_091133
KEY
BLACK HOLE
NEUTRON STAR
(SHOWN AT X10 SCALE)
UNCERTAIN OBJECT
PRIMARY MASS
SECONDARY MASS
FINAL MASS
DATE(_TIME)
31 1.2 32 GW191219_163120
UNITS ARE SOLAR MASSES
1 SOLAR MASS = 1.989 x 10^30 kg
GRAVITATIONAL WAVE
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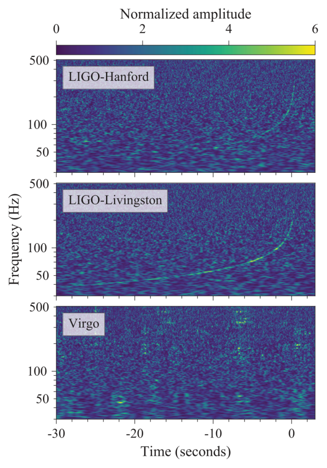
GW170817

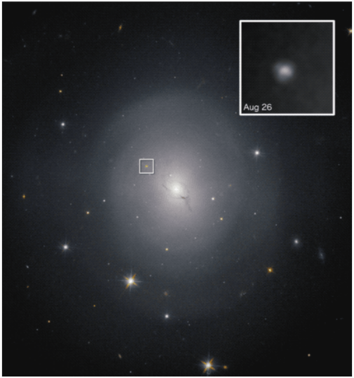
1. August 17, 2017: Binary neutron star inspiral + merger



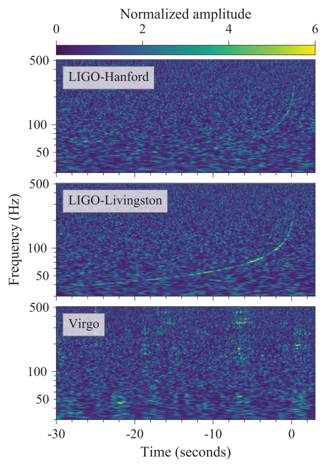
(Artists impression; LIGO-Virgo-KAGRA)

GW170817

1. August 17, 2017: Binary neutron star inspiral + merger
2. Merger: burst of electromagnetic radiation



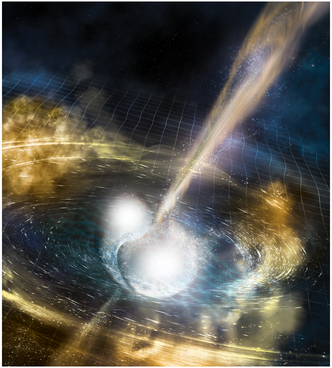
(NASA)



(Artists impression; LIGO-Virgo-KAGRA)

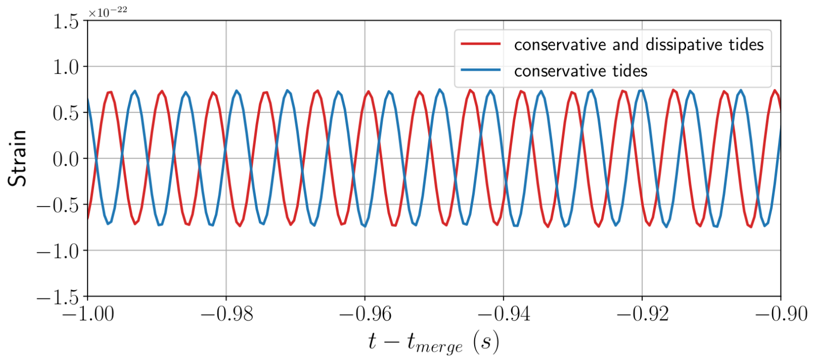
GW170817: facts

1. Masses: $\sim 1.4\mathrm{M}_{\odot}$, $\sim 1.3\mathrm{M}_{\odot}$
2. Signal duration: $\sim 2\ \mathrm{min}$



(Artists impression; LIGO-Virgo-KAGRA)

Constraints on $\bar{\Lambda}$ and $\bar{\Xi}$ from GW170817



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Gravitational wave data analysis

Bayes' rule/theorem:

$$P(\boldsymbol{\theta}|\boldsymbol{d}) = \frac{P(\boldsymbol{d}|\boldsymbol{\theta})\, P(\boldsymbol{\theta})}{P(\boldsymbol{d})}.$$

1. $\boldsymbol{d} \sim h$: Measured strain in detectors

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4. `Bilby` (Ashton+ 2018) with nested sampling library `dynesty` (Speagle 2019)

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$$P(\boldsymbol{\theta}|\boldsymbol{d}) = \frac{P(\boldsymbol{d}|\boldsymbol{\theta})\,P(\boldsymbol{\theta})}{P(\boldsymbol{d})}.$$

1. $\boldsymbol{d} \sim h$: Measured strain in detectors
2. $\boldsymbol{\theta}$: $\left(m_A, m_B, D, ..., \bar{\Lambda}, \bar{\Xi}\right)$
3. $P(\boldsymbol{d}|\boldsymbol{\theta}) \sim S_n(h, h(\boldsymbol{\theta}; f))$: $h(\boldsymbol{\theta}\ f)$: `IMRPhenomPv2_NRTidal` (Dietrich+ 2018) + $\bar{\Xi}$ phase correction
4. `Bilby` (Ashton+ 2018) with nested sampling library `dynesty` (Speagle 2019)

What is $P\left(\bar{\Lambda}|d\right)$ from GW170817?

What is $P\left(\bar{\Xi}|d\right)$ from GW170817?

Old: $P\left(\bar{\Lambda}\right)$ for GW170817

$$\Delta\Psi_\Lambda = -\frac{117}{256}\frac{1}{\eta_{SM}}\bar{\Lambda}u^5 + \cdots .$$

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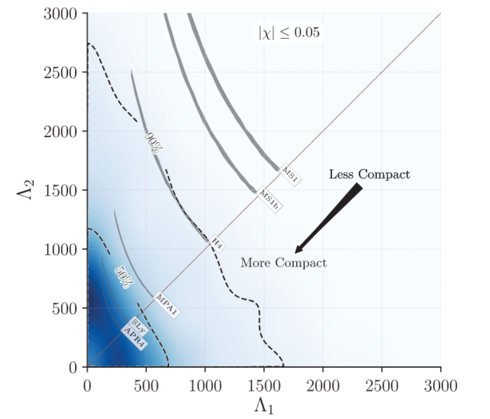

LIGO/Virgo 1710.05832

Old: $P\left(\bar{\Lambda}\right)$ for GW170817

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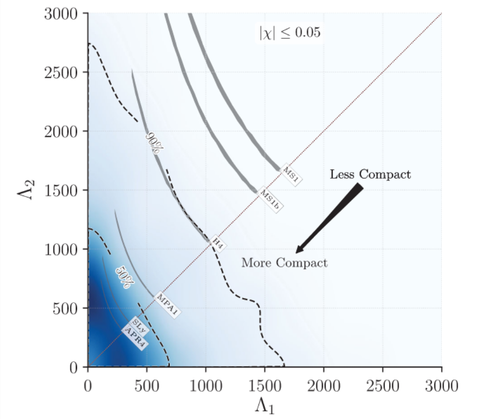


LIGO/Virgo 1710.05832

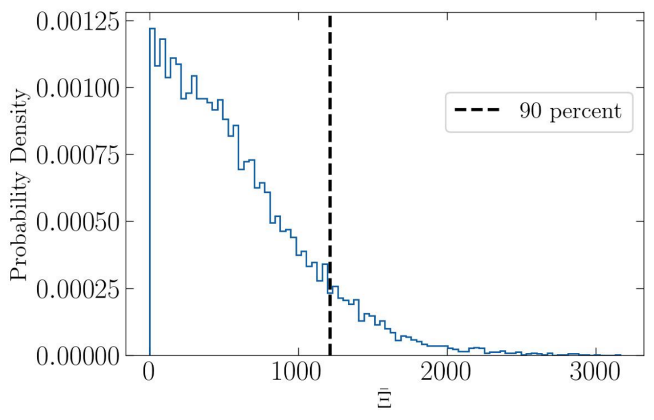
Multiple equations of state $p(\rho)$ ruled out/disfavored by data.

New: $P\left(\Xi\right)$ for GW170817

$$\Delta\Psi_{\Xi} = -\frac{225}{4096}\frac{1}{\eta_{SM}}\Xi u^3 \log\left(u\right).$$

New: $P\left(\bar{\Xi}\right)$ for GW170817

$$\Delta\Psi_{\bar{\Xi}} = -\frac{225}{4096}\frac{1}{\eta_{SM}}\bar{\Xi}u^3\log\left(u\right).$$



2312.11659 JLR, Hegade, Chandramouli, Yunes

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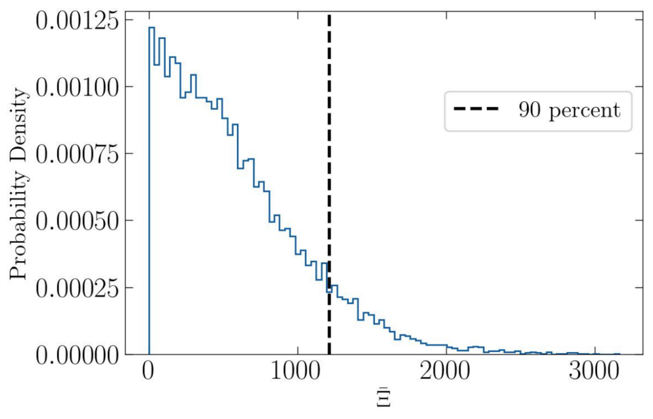
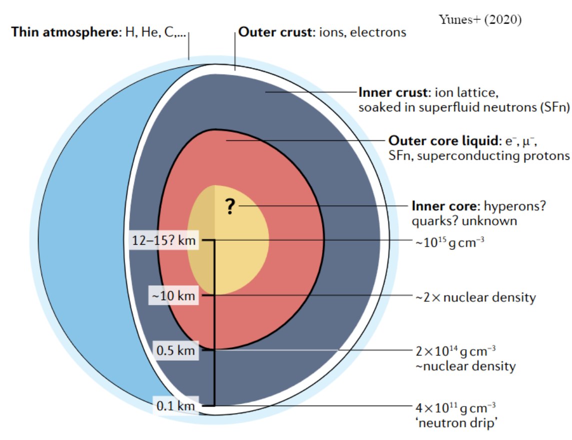


2312.11659 JLR, Hegade, Chandramouli, Yunes

$$\langle\zeta\rangle \lesssim 10^{31}\,\frac{\mathrm{g}}{\mathrm{cm\ s}} \qquad \langle\eta\rangle \lesssim 10^{28}\,\frac{\mathrm{g}}{\mathrm{cm\ s}}$$

PRELIMINARY: Implications for nuclear theory

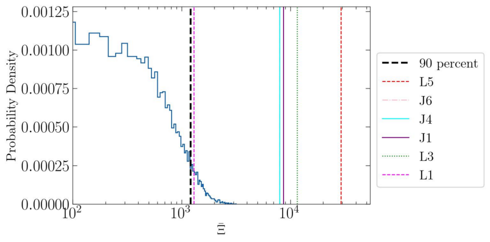
PRELIMINARY



PRELIMINARY

24–.— Hegade, Yang, Teixera, Noronha, Noronha-Hostler, Yunes, JLR, ... (work in progress...)

PRELIMINARY: Implications for nuclear theory



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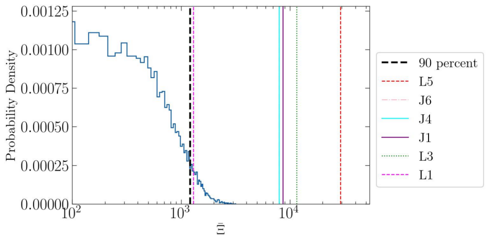

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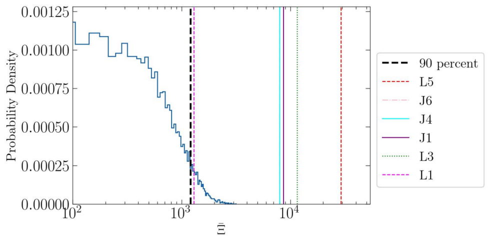

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CAUTION: Temperature of stars unknown

$$\zeta_{Urca} \propto T^4 \qquad \zeta_{mUrca} \propto T^6 \qquad \zeta_{Hyperon} \propto T^{-2}$$

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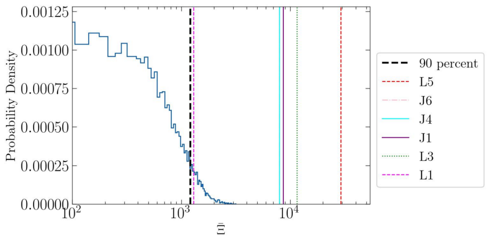

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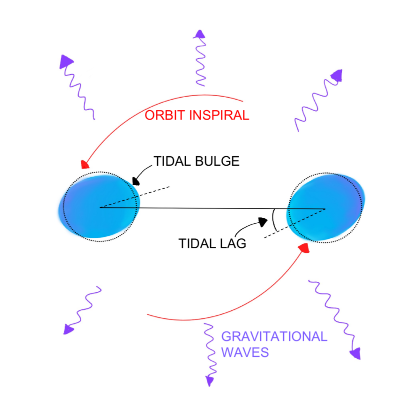
$$\zeta_{Urca} \propto T^4 \qquad \zeta_{mUrca} \propto T^6 \qquad \zeta_{Hyperon} \propto T^{-2}$$

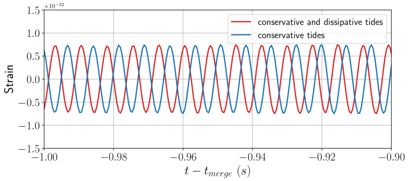
CAUTION: more work on theory prediction for ζ, η

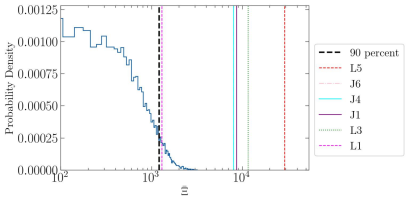
$$\zeta(\omega)$$

Putting everything together

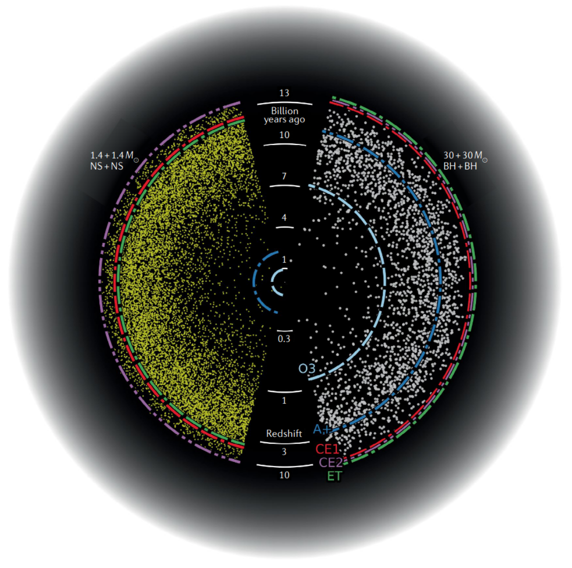
1. $\Psi \to \bar{\Lambda} \to \bar{\Xi} \to p(\rho),\, \zeta(\rho),\, \eta(\rho) \to$ nuclear physics
2. Can probe equilibrium & out-of-equilibrium nuclear physics of neutron stars





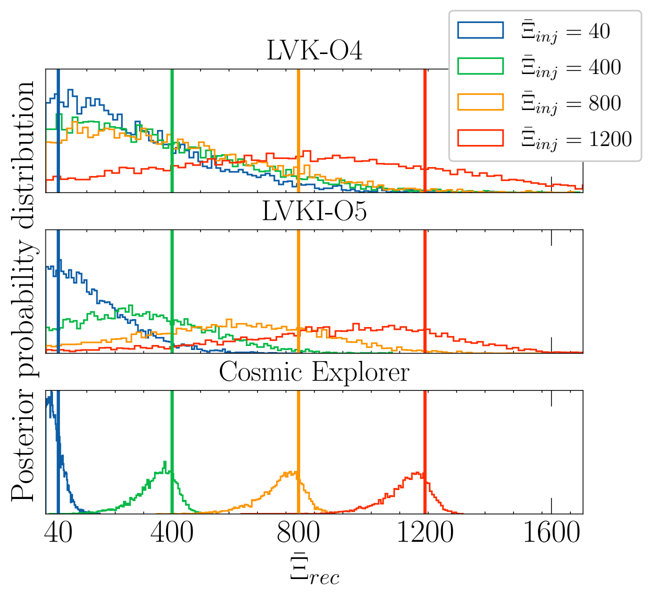


Better detectors → more binary neutron star detections



Bailes+ 2021

Better detectors → better constraints



Future work

- Compute *relativistic*, disspative tidal response (Hegade, JLR, Yunes)
- Relativistic, dissipative tidal response for physically relatistic equations of state (Hegade, Yang, ...)
- Temperature evolution of the stars (Lai 1993; Saes, Hegade)
- Higher PN corrections to gravitational wave phase (Hegade)

Conclusions

- Neutron stars: densest objects in the universe
- Tidal deformability: reflect the material properties of neutron stars
- Can probe equilibrium and *out-of-equilibrium* physics with gravitational waves
- Other work
  - Mass vs. radius and $p(\rho)$ (Riley+ 2021, ...)
  - Other non-equilibrium properties (oscillations of stars) (Steinhoff+ (2016), Pratten+ (2019))
  - Quasi-universal neutron star relations (Steinhoff+ (2016), Pratten+ (2019))

Backup slides

More details

Self-consistency of Newtonian calculation: no-spin calculation

- Tidal torquing spins-up stars

$$\frac{d\Omega_A}{dt} \approx \frac{45 m_B^2 m_A^5}{2 R_A^2 M^6} \Xi_A \gamma_0^6 \left(\omega_A - \Omega_A\right).$$

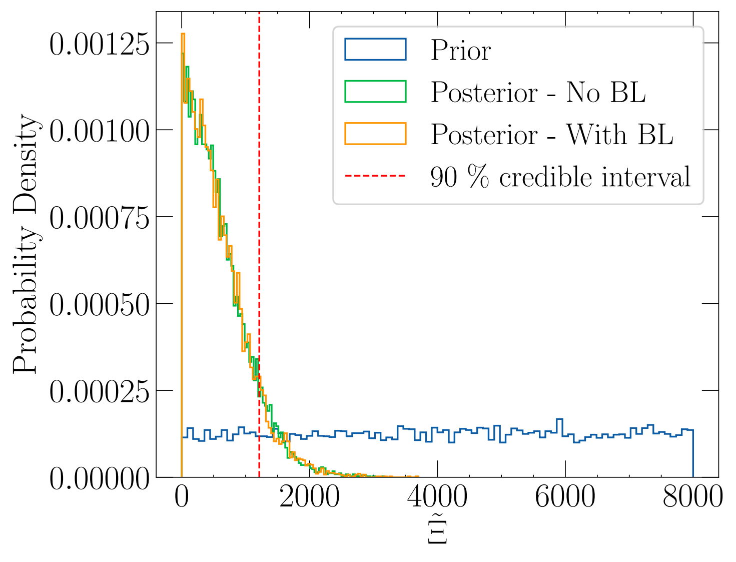
- Inspiral driven by gravitational radiation reaction

$$\frac{dr}{dt} \approx -\frac{64 \eta_{SM}}{M} \gamma_0^4.$$

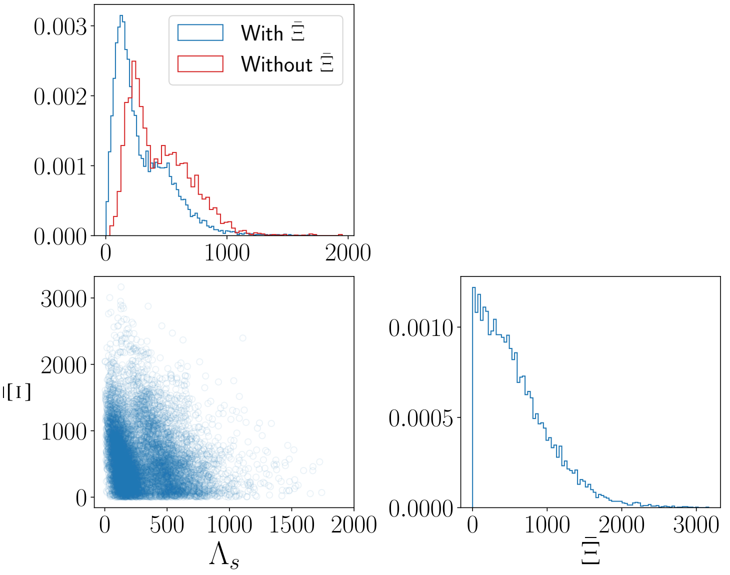
- $\nu \gtrsim$ average causal bound in order for appreciable spinup of stars (tidal locking) before merger (Bildsten+Cutler 1992)

$$\frac{T_{lock}}{T_{insp}} \approx 10^2 \left(\frac{M}{3.2 M_\odot}\right)^3 \left(\frac{m_A}{1.6 M_\odot}\right)^3 \left(\frac{1.6 M_\odot}{m_B}\right) \left(\frac{12 km}{R_A}\right)^5 \left(\frac{10^{16}\mathrm{cm}^2\mathrm{s}^{-1}}{\nu}\right) \times \left(\frac{0.1}{p_{2,A}}\right) \left(\frac{0.1}{k_{2,A}}\right)$$

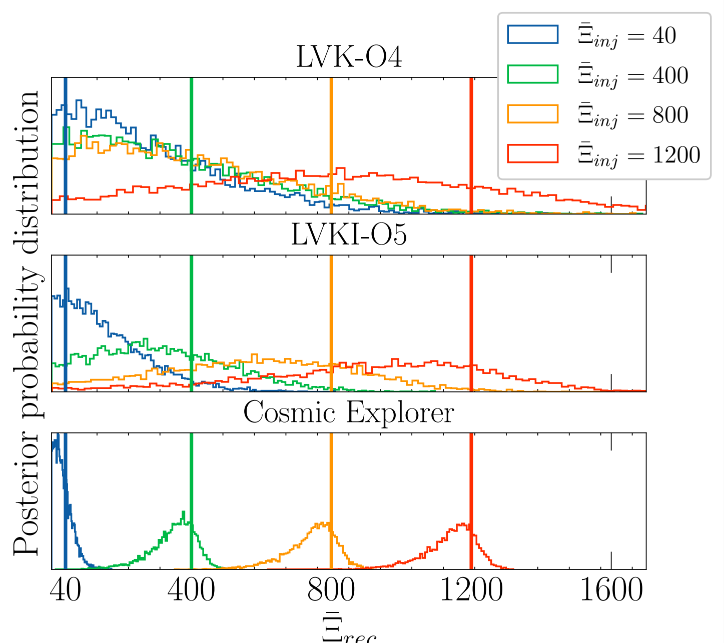
More details on constraint



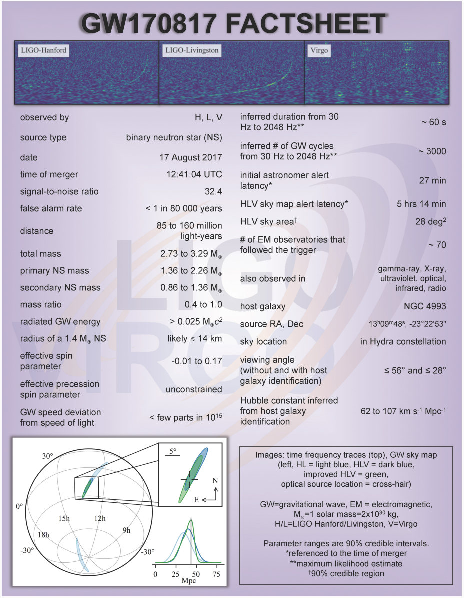
Correlation between Λ and Ξ



Future constraints



GW170817 fact sheet



GW170817 FACTSHEET

observed by	H, L, V
source type	binary neutron star (NS)
date	17 August 2017
time of merger	12:41:04 UTC
signal-to-noise ratio	32.4
false alarm rate	< 1 in 80 000 years
distance	85 to 160 million light-years
total mass	2.73 to 3.29 $M_\odot$
primary NS mass	1.36 to 2.26 $M_\odot$
secondary NS mass	0.86 to 1.36 $M_\odot$
mass ratio	0.4 to 1.0
radiated GW energy	> 0.025 $M_\odot c^2$
radius of a 1.4 $M_\odot$ NS	likely ≲ 14 km
effective spin parameter	-0.01 to 0.17
effective precession spin parameter	unconstrained
GW speed deviation from speed of light	< few parts in 10^{15}

inferred duration from 30 Hz to 2048 Hz**	~ 60 s
inferred # of GW cycles from 30 Hz to 2048 Hz**	~ 3000
initial astronomer alert latency*	27 min
HLV sky map alert latency*	5 hrs 14 min
HLV sky area†	28 deg^2
# of EM observatories that followed the trigger	~ 70
also observed in	gamma-ray, X-ray, ultraviolet, optical, infrared, radio
host galaxy	NGC 4993
source RA, Dec	$13^h09^m48^s$, -23°22'53"
sky location	in Hydra constellation
viewing angle (without and with host galaxy identification)	≲ 56° and ≲ 28°
Hubble constant inferred from host galaxy identification	62 to 107 km s^{-1} Mpc^{-1}

Images: time frequency traces (top), GW sky map (left, HL = light blue, HLV = dark blue, improved HLV = green, optical source location = cross-hair)

GW=gravitational wave, EM = electromagnetic, $M_\odot$=1 solar mass=2×10^{30} kg, H/L=LIGO Hanford/Livingston, V=Virgo

Parameter ranges are 90% credible intervals.
*referenced to the time of merger
**maximum likelihood estimate
†90% credible region

Figure 1: LVK

Can we relate tides to microphysics?

- Calculations of Ξ for large planets/stars often very different from what is measured (Ogilvie 2014)

Apollo 17/NASA

- Ocean: $\sim 0.023\%$ of the Earth's total masss
- $\sim 95\%$ tidal dissipation of Earth: from the ocean (Auclair-Desrotour+ 2018)

Calculating the tidal response: black holes

$$Q_{\mu\nu}(\omega) = F_2(\omega)\, E_{\mu\nu}(\omega)\,,$$

- Tidal response of black holes (Poisson 2009)

$$F_2(\omega) \sim 10^{-6} s \left(\frac{M}{20 M_\odot}\right) i\omega + \mathcal{O}\left(\omega^2\right).$$

Computing the GW phase

- GW strain h

$$h(f) = A(f)\, e^{i\Psi(f)}.$$

- Differential equation for phase in terms of total binding energy E_{tot} of a binary (Tichy+ 2000)

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

- Compute $E_{tot}, \dot{E}_{tot}$ for a Newtonian binary, including keeping dissipative tidal response

Effacement principle

$$Q_{ij} = -\Lambda E_{ij} - \Xi \partial_t E_{ij}.$$

- **Effacement principle:** finite size of star affects equations of motion at **5PN** order (Damour 1987).
  - Dissipative finite size of star effects enter equations of motion at **6.5PN** order
- Q: How do dissipative, finite size effects enter at **4PN** in the GW phase?
- A: Dissipation affects $\dot{E}_{tot}$ through $\mathcal{F}_{diss}$

$$\frac{d^2\Psi}{df^2} = \frac{2\pi}{\dot{E}_{tot}} \frac{dE_{tot}}{df}.$$

Adiabatic Love numbers

1. Spherically symmetric star (TOV equations)

$$g^{(0)}_{\mu\nu},\ \delta u^{\mu}_{(0)},\ \rho_{(0)},\ \ldots\ \rightarrow\ G^{(0)}_{\mu\nu} = \frac{8\pi G}{c^4} T^{(0)}_{\mu\nu}.$$

2. **Adiabatic** (time independent) linear perturbation (Hinderer 2008, Damour+Nagar 2009, Binnington+Poisson 2009)

$$\delta g_{\mu\nu},\ \delta u^{\mu},\ \delta\rho,\ \ldots\ \rightarrow\ \delta G_{\mu\nu} = \frac{8\pi G}{c^4}\delta T_{\mu\nu}.$$

3. Extract quadrupole from g_{tt} (Thorne 1998, Hinderer 2008)

No viscous corrections to stress-energy tensor

1. Perturbed stress energy reduces to perfect fluid.

$$\delta T_{\mu\nu} = \delta\left(\frac{e}{c^2} u_\mu u_\nu + p\Delta_{\mu\nu}\right).$$

2. There are no viscous corrections to the adiabatic tidal Love numbers.
3. Can extend argument to other fluid models.

Relativistic, viscous fluids

Relativistic, causal, hyperbolic theory of viscous fluids: **BDNK fluid** (Kovtun 2019, Bemfica+ 2020)

$$\begin{aligned}
T_{\mu\nu} =& \mathcal{E} u_\mu u_\nu + \mathcal{P}\Delta_{\mu\nu} + \boxed{2\mathcal{Q}_{(\mu}u_{\nu)}} - \boxed{2\eta\,\sigma_{\mu\nu}}, \\
\mathcal{E} \equiv& \frac{1}{c^2}\left(e + \boxed{\tau_\epsilon\left[u^\alpha\nabla_\alpha e + (e+p)\,\theta\right]}\right), \\
\mathcal{P} \equiv& p - \boxed{\zeta\theta} + \boxed{\tau_p\left[u^\alpha\nabla_\alpha e + (e+p)\,\theta\right]}, \\
\mathcal{Q}_\mu =& \boxed{\tau_Q\left(\frac{(e+p)}{c^2}u^\alpha\nabla_\alpha u_\mu + \Delta_\mu^\alpha\nabla_\alpha p\right)} \\
&+ \boxed{\frac{\rho\kappa T^2}{m_b(e+p)c^2}\Delta_\mu^\alpha\nabla_\alpha\left(\frac{\mu}{T}\right)}..
\end{aligned}$$

More facts about the BDNK fluid model

- **BDNK fluid** (Kovtun 2019, Bemfica+ 2020) is the **only** relativistic fluid model that
    1. Is causal and strongly hyperbolic.
    2. Has stable equilibrium states.
    3. Includes bulk viscosity, shear viscosity, and heat conduction.
    4. Includes nonzero baryon number.
    5. Entropy increases with time.

More facts about the BDNK fluid model

1. Fluid current

$$J^{\mu} = \rho u^{\mu}.$$

2. Set $\tau_{\epsilon} = 0$, $\tau_p = 0$, $\tau_Q = 0$, and the theory reduces to an **Eckart fluid** (Eckart 1940).
3. Requiring that the theory by hyperbolic in the relativistic regime, and reduce to the Navier-Stokes equations of motion at 0PN constrains the heat conductivity and shear viscosity to satisfy $\kappa > \eta k_B / m_b$ (Hegade+ 2023).