Evolution of binary black hole systems in scalar Gauss-Bonnet gravity [1]

Seminar, Albert Einstein Institute

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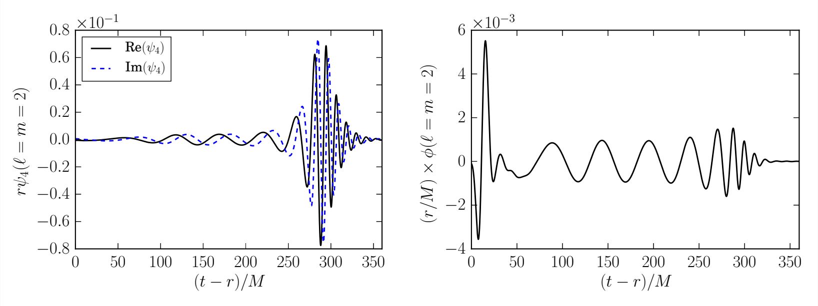
[1] arXiv:2011.03547, arXiv:2105.08571

Outline and Summary

$$S = \int d^4x\sqrt{-g}\left[R - (\nabla\phi)^2 + \lambda\phi\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}\right)\right]$$



Goals: understand why we choose to study binary black hole systems in this theory, and understand how we generated these waveforms.

Outline and Summary

1. Testing General Relativity (GR) with gravitational waves
2. Model-dependent tests of GR: Einstein scalar Gauss-Bonnet (ESGB) gravity
3. Numerical simulation of black holes spacetimes in ESGB gravity
4. If time/interest: formulation of equations of motion that allows for numerical evolution of ESGB gravity

Outline

Gravitational wave astronomy

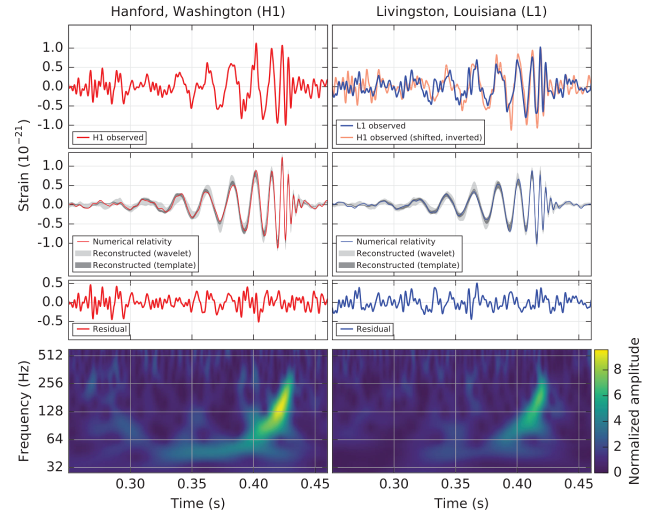


Figure: Observation of Gravitational Waves from a Binary Black Hole Merger, *Phys.Rev.Lett.* 116, 061102 (2016)

To characterize the physics that produced measured GW signals, need templates

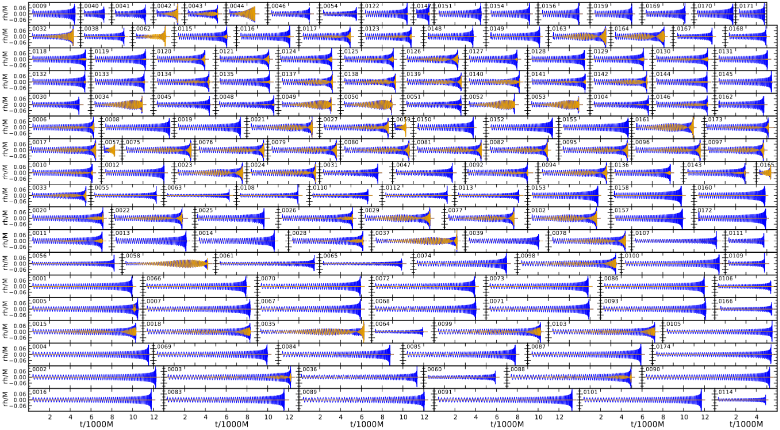
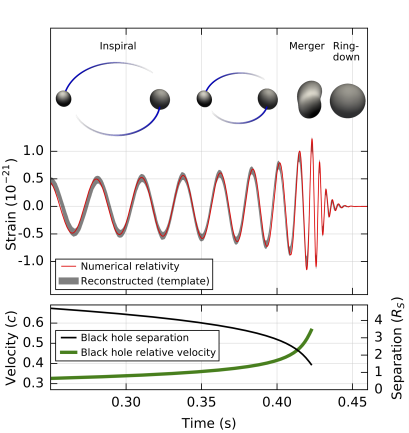


FIG. 3: Waveform polarizations $(r/M)h_+$ (blue) and $(r/M)h_\times$ (orange) in a sky direction parallel to the initial orbital plane of each simulation. The unit of the time axis corresponds to $1000M = 0.1s$ for binaries with total mass $M = 20M_\odot$.

Figure: A catalog of 174 binary black-hole simulations for gravitational-wave astronomy, *Phys.Rev.Lett.* 111 (2013) 24, 241104

Model independent (consistency) tests

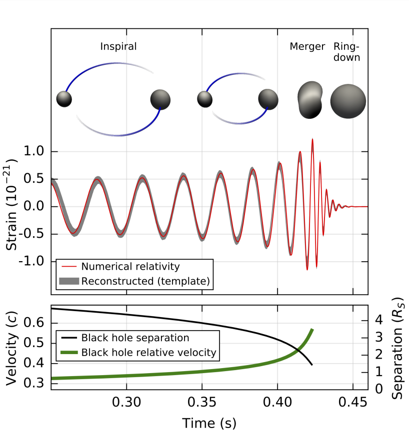


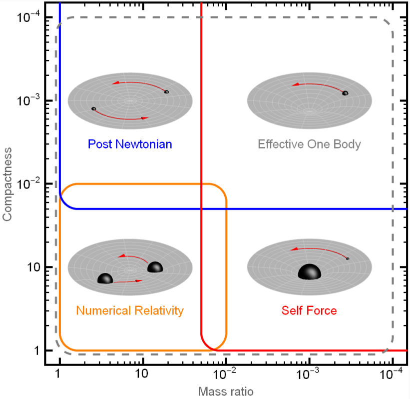
Compare GR predictions for inspiral, merger ringdown, and check if all give the same answer

Model dependent tests

- Need predictions from modified gravity theories to map measured deviations from GR to a physical process
  - **Black hole hair**
  - New light fields
  - High curvature terms in the action
  - ...

Numerical relativity for modified theories of gravity





To fully leverage the power of GW to constrain (discover?) potential modifications of GR, need to understand the merger phase of binary black hole evolution $\Longrightarrow$ need **numerical relativity for modified gravity**.

Outline

Guiding principles

We want to look at a classical field theory that

1. Has a mathematically sensible interpretation
2. Matches all current (non gravitational-wave) observations
3. Has novel black hole phenomenology (e.g. black hole hair)
4. (Optional): is "natural", e.g. appears in low-energy effective actions of some UV completion of GR

Einstein scalar Gauss-Bonnet (ESGB) gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\left(\nabla\phi\right)^2 - V\left(\phi\right) + \beta\left(\phi\right)\mathcal{G}\right),$$

where $\mathcal{G}$ is the *Gauss-Bonnet scalar*

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}.$$

ESGB gravity

1. Has a mathematically sensible interpretation[2]
2. Relatively unconstrained by current observations[3]
3. Black holes in variants of ESGB gravity have scalar "hairy" black holes solutions[4]
4. The term $\beta(\phi)\mathcal{G}$ captures leading order scalar-tensor parity invariant interactions, so appears in the leading order corrections for many UV complete theories of gravity[5]

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[2] Provided the modified gravity corrections are "small" e.g. JR & Pretorius, Class.Quant.Grav. 36 (2019) 13, 134001, Kovacs et. al. Phys.Rev.D 101 (2020) 12, 1240030

[3] Baker et. al. Phys.Rev.Lett. 119 (2017) 25, 251301, Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010, Perkins et. al. Phys.Rev.D 104 (2021) 2, 024060, Saffer and Yagi arXiv:2110.02997

[4] e.g. Sotiriou and Zhou Phys.Rev.D 90 (2014) 124063

[5] e.g. Kovacs and Reall Phys.Rev.Lett. 124 (2020) 22, 221101

Outline

Testing GR with gravitational waves

Einstein scalar Gauss-Bonnet gravity

Numerical simulation of black hole spacetimes in ESGB gravity

Conclusion

Shift symmetric ESGB gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\left(\nabla\phi\right)^2 + \lambda\phi\mathcal{G}\right),$$

This theory does not admit **stationary** Schwarzschild black hole solutions[6]; instead "hairy" scalar black holes should be end states in this theory.

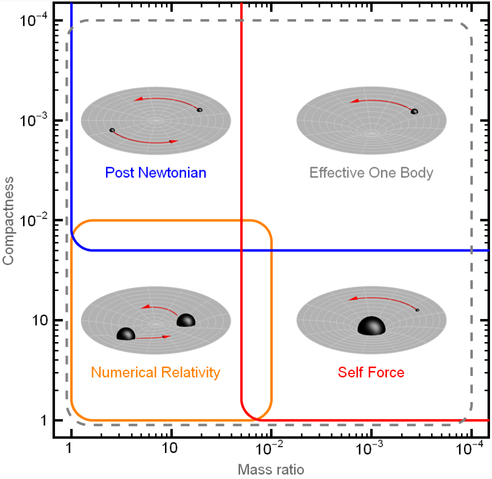
$$\Box\phi + \lambda\mathcal{G} = 0$$

Amount of scalar hair $\sim \lambda/m^2$, so for a fixed λ, the modified gravity effects are larger for smaller black hole masses m.

[6]Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

Our approach to studying ESGB gravity

Study exact (nonperturbative) solutions to the theory, to model the merger phase of binary black hole evolution.



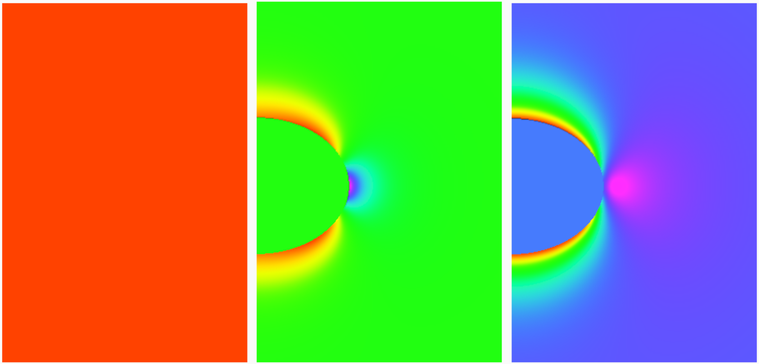
Black hole spacetimes in shift symmetric ESGB gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R - \frac{1}{2}\left(\nabla\phi\right)^2 + \lambda\phi\mathcal{G}\right),$$

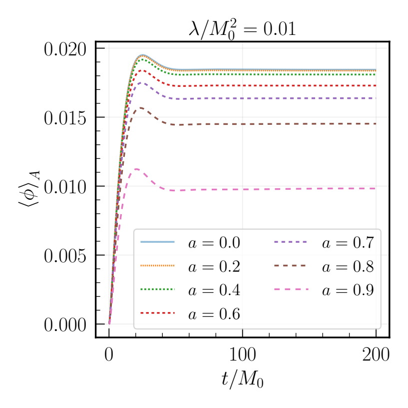
Based on Will East and JR, *Phys.Rev.D* 103 (2021) 4, 044040, arXiv:2011.03547

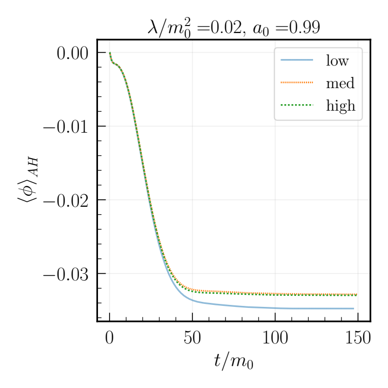
1. Reformulate the equations of motion in *modified generalized harmonic* formulation
2. Spinning black hole evolution (axisymmetric spacetime)
3. Head on black hole collisions (axisymmetric spacetime)
4. Binary black hole merger (no symmetry assumptions)

1. (Spinning) black hole evolution



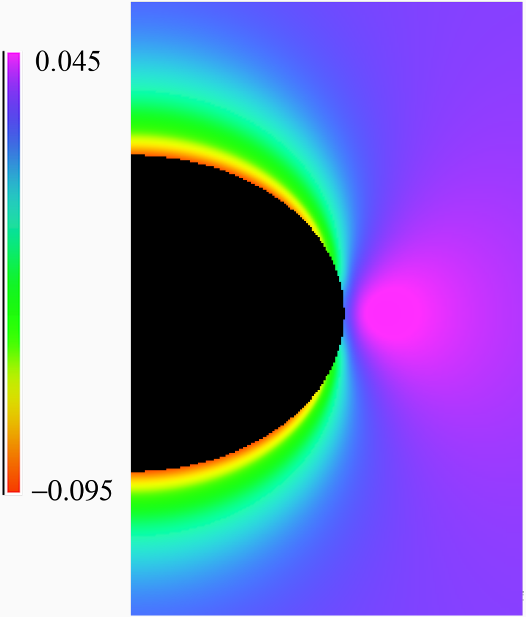
Scalar hair growth around spinning black holes



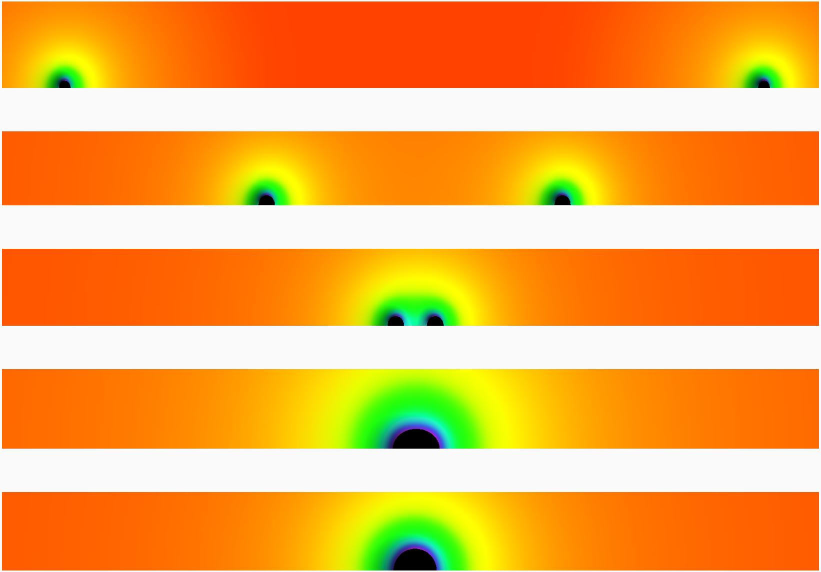


- $\langle\phi\rangle_A$: average scalar field value on black hole horizon
- a: initial dimensionless black hole spin

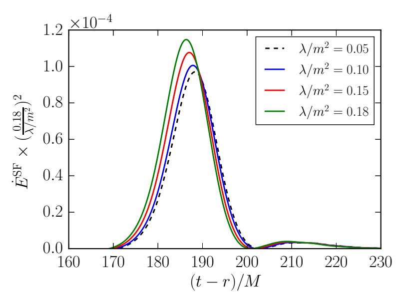
Scalar field density around a spinning black hole

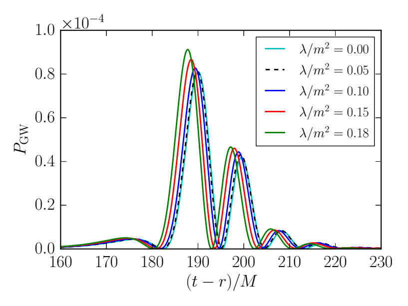


2. Head on black hole collisions



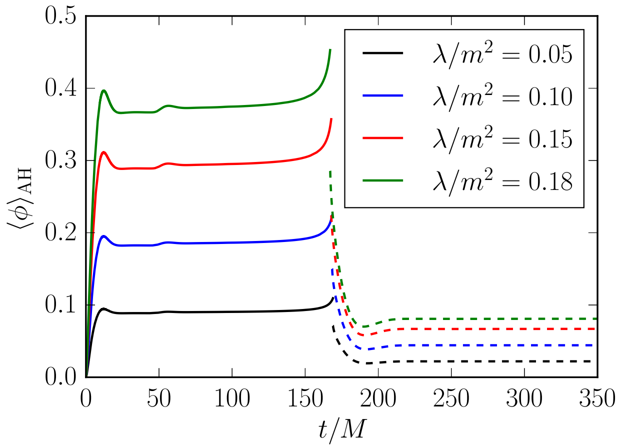
Head on black hole collisions: gravitational and scalar radiation





Flux of scalar field vs flux of gravitational waves

Head on black hole collisions: scalar field on horizon



$$\langle\phi\rangle_{AH} \sim \lambda/m^2$$

3. Binary black hole collisions

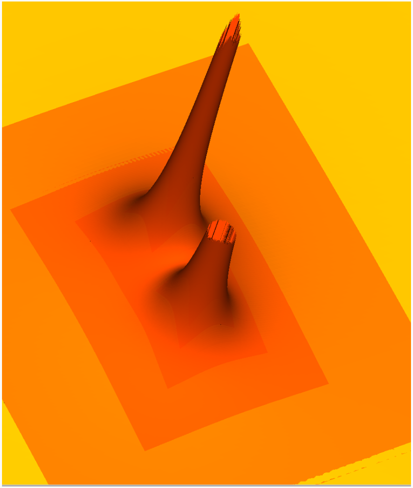


Figure: Scalar field about two non-spinning black holes, 3:2 mass ratio, $\lambda/m_1^2 = 0.01$

Gravitational wave strain from two ESGB binary black holes merging

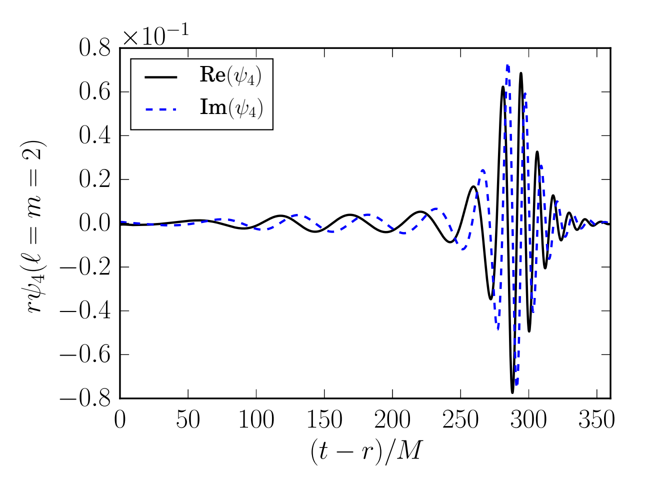


Figure: Non-spinning black holes, equal mass ratio, $\lambda/m^2 = 0.01$

Scalar waves from two ESGB binary black holes merging

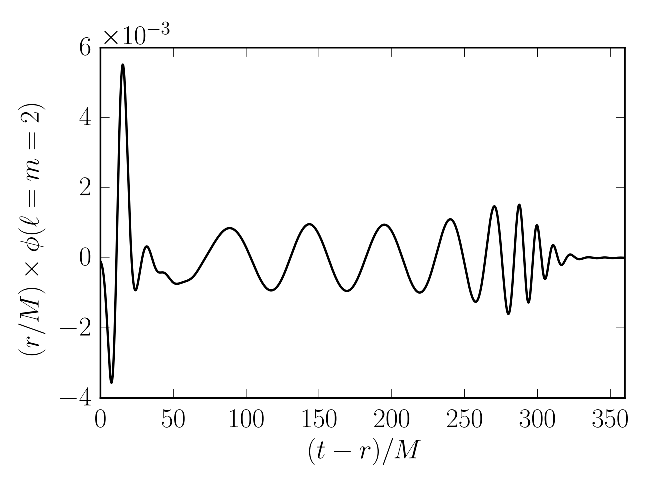


Figure: Non-spinning black holes, equal mass ratio, $\lambda/m^2 = 0.01$

Binary black hole spacetime: convergence of constraint violation

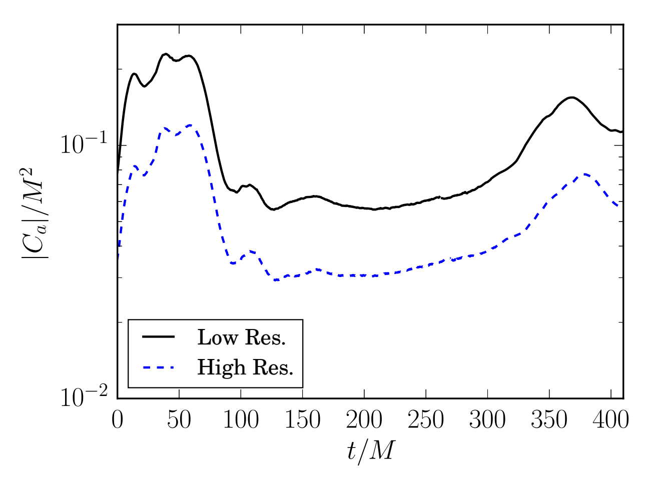


Figure: Non-spinning black holes, equal mass ratio, $\lambda/m^2 = 0.01$.
$C^\alpha \equiv H^\alpha + \tilde{g}^{\mu\nu}\Gamma^\alpha_{\mu\nu}$

Other ESGB gravity theories

$$S = \int dx^4 \sqrt{-g} \left(\frac{1}{2} R - \frac{1}{2} (\nabla\phi)^2 + \left(\frac{\lambda}{2}\phi^2 + \frac{\sigma}{4}\phi^4 \right) \mathcal{G} \right).$$

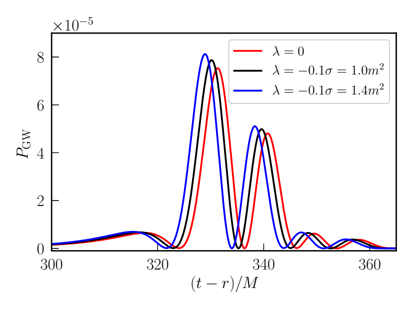
- GR black holes are stationary solutions

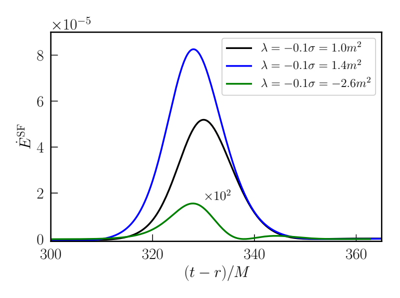
$$\Box\phi + \left(\lambda + \sigma\phi^3\right) \phi\mathcal{G} = 0.$$

- Depending on sign and size of λ, σ, GR black holes can be unstable to forming scalarized black holes from small scalar field perturbations[7]
- Need to start with slightly perturbed GR black holes to see "interesting" dynamics

[7] e.g. Silva et al Phys. Rev. Lett. 120, 131104, Doneva et. al. Phys. Rev. Lett. 120, 131103 (2018), Minamitsuji and Ikeda Phys. Rev. D 99, 044017 (2019)

Other ESGB gravity theories





Flux of gravitational and scalar radiation for equal mass head-on collisions (from East and JR Phys.Rev.Lett. 127 (2021) 10, 101102)

Modified generalized harmonic (MGH) formulation[8]

- Specify two auxiliary Lorentzian metrics $\hat{g}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ in addition to the spacetime metric $g^{\mu\nu}$
- Specify the gauge/coordinate condition with:

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu x^\gamma = H^\gamma,$$

  where H^γ is source function
- Free parameters: $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$, H^γ (more details given at end of talk)
- Besides using the MGH formulation, we begin with GR initial data, and use standard techniques from numerical relativity

[8] Kovacs and Reall, Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

MGH formulation

- ESGB gravity has a well-posed initial value problem in generic spacetimes, provided the modified gravity corrections are "small", when one specifies their coordinate according to a *modified generalized harmonic* (MGH) condition[9]:

$$H^{\gamma} + \Gamma^{\gamma}_{\alpha\beta}\tilde{g}^{\alpha\beta} = 0.$$

- H^{γ}: free function one can choose
- $\tilde{g}^{\alpha\beta}$: "auxiliary" metric one can choose (*not* the "physical" metric $g^{\alpha\beta}$)
- In contrast to "generalized harmonic" formulation[10]: $H^{\gamma} + \Gamma^{\gamma}_{\alpha\beta}g^{\alpha\beta} = 0$

[9] Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

[10] e.g. Pretorius, Class.Quant.Grav. 22 (2005) 425-452

MGH formulation

- Degrees of freedom propagate with different characteristic speeds in the MGH formulation
  - "Pure gauge modes": propagate along null cone of $\tilde{g}^{\mu\nu}$
  - "Gauge violating modes": propagate along null cone of $\hat{g}^{\mu\nu}$
  - "Physical modes": propagate along null cone of $g^{\mu\nu}$
- In, e.g. generalized harmonic formulation, all of these modes propagate along null cone of $g^{\mu\nu}$

$$0 = \tilde{g}^{\alpha\beta}\nabla_\alpha\nabla_\beta x^\gamma + \cdots$$
$$0 = \hat{g}^{\alpha\beta}\nabla_\alpha\nabla_\beta C^\gamma + \cdots$$

Outline

Conclusion

- Characterizing potential modifications of general relativity in binary black hole spacetimes requires having accurate templates
- Understanding the merger phase of binary black hole evolution in modified gravity theories requires numerical relativity methods
- **Claim:** We now have the tools to produce gravitational waveforms produced during the merger of two black holes for a class of modified gravity theories that have black hole hair

Future directions

- Binary black hole inspiral
  1. Different mass ratios
  2. Different couplings λ
  3. Different forms of scalar field potential V and Gauss-Bonnet coupling β (e.g. theories which have "spontaneous scalarization")
- Develop accurate, low initial eccentricity initial data
- Characterize degeneracies in parameter space vs. GR
- Compare "exact" (numerical) solutions to approximate solutions
  1. Order-reduction methods
  2. Post-Newtonian calculations
  3. Black hole quasinormal mode excitation factors
- Further develop the MGH formulation of general relativity and scalar-tensor gravity theories (e.g. What are "good" choices for the auxiliary metrics?)

Backup slides

Why does MGH formulation work?

- Can show ESGB gravity is hyperbolic, at least when $\beta(\phi)\mathcal{G}$ is small[11]
- The difficult part is to find a *strongly* hyperbolic formulation of the theory
- Example of matrix with real eigenvalues, no complete set of eigenvectors

$$\mathcal{P}_{GH} \sim \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

[11] e.g. Papallo and Reall Phys.Rev.D 96 (2017) 4, 044019

Why does MGH formulation work? Modified characteristic speeds

- Insight of Kovacs and Reall: change the characteristic speeds of gauge and constraint violating degrees of freedom to make principal symbol have a complete set of eigenvectors

$$\mathcal{P}_{GH} \sim \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \qquad \lambda = \{1, 1\} \qquad \vec{v} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$$

$$\mathcal{P}_{MGH} \sim \begin{pmatrix} 1 & 2 \\ 0 & 1.1 \end{pmatrix} \qquad \lambda = \{1, 1.1\} \qquad \vec{v} = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.998752 \\ 0.0499376 \end{pmatrix} \right\}$$

- "Pure gauge modes": propagate along null cone of $\tilde{g}^{\mu\nu}$
- "Gauge violating modes": propagate along null cone of $\hat{g}^{\mu\nu}$
- "Physical modes": propagate along null cone of $g^{\mu\nu}$

Why does MGH formulation work? Modified characteristic speeds

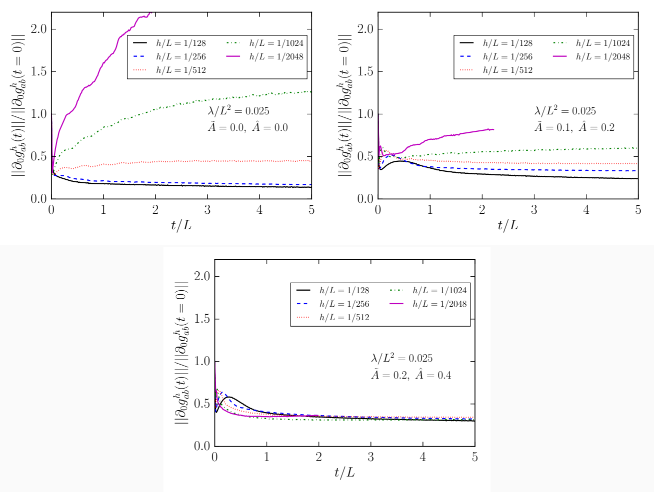
- Generalized harmonic

$$0 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta x^\gamma + \cdots$$
$$0 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta C^\mu + \cdots$$

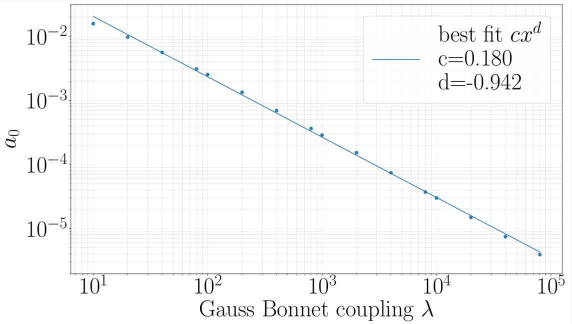
- Modified generalized harmonic

$$0 = \tilde{g}^{\alpha\beta}\nabla_\alpha\nabla_\beta x^\gamma + \cdots$$
$$0 = \hat{g}^{\alpha\beta}\nabla_\alpha\nabla_\beta C^\gamma + \cdots$$

Hyperbolicity test: Self-convergence in harmonic vs modified harmonic gauge



Limits of Gauss-Bonnet coupling: breakdown of hyperbolicity



- At large enough Gauss-Bonnet couplings, shift-symmetric ESGB gravity loses hyperbolicity[12]

$$\phi_0 = a_0 \left(\frac{r}{w_0}\right)^2 \exp\left(-\left(\frac{r-r_0}{w_0}\right)^2\right).$$

[12] e.g. JR and Pretorius Phys.Rev.D 99 (2019) 8, 084014

Planck units

- (reduced) Planck units: $8\pi G = c = \hbar = k_B = 1$
- Everything can be phrased in terms of the *geometrized dimension L*
- Energy scale, etc. are multiples of:
  - Planck energy: $E_p = l_p c^4/G \sim 10^{16} ergs \sim 10^{19} GeV$
  - Planck length: $l_p = (G\hbar/c^3)^{1/2} \sim 10^{-33} cm$
  - Planck time: $t_p = l_p/c \sim 10^{-44} s$
  - Planck mass: $m_p = l_p c^2/G \sim 10^{-5} g$
  - Planck temperature $E_p/k_B \sim 10^{32} K$