General relativity and its classical modification in gravitational collapse

Final Public Oral

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Nonlinear dynamics of the modified gravity theory *Einstein-dilaton Gauss-Bonnet (EdGB) gravity*

Theory of gravity that has a spacetime metric $g_{\mu\nu}$ and a "dilaton" (real scalar field) ϕ

$$S_{EdGB} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left(R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - W(\phi) \mathcal{G} \right), \tag{1}$$

where $\mathcal{G}$ is the (four dimensional) *Gauss-Bonnet scalar*

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}. \tag{2}$$

Results

- Numerically studied EdGB gravity in spherical symmetry
- Computed dilaton dynamics and ‘scalarized’ black hole solutions
- Found EdGB gravity solutions can violate the Null Convergence Condition
- Found a break down in hyperbolicity of the theory for sufficiently large modified gravity corrections: EdGB gravity is fundamentally ‘mixed type’

Why is EdGB a modified gravity theory?

- EdGB gravity is a "modified" gravity theory as it is not General Relativity (GR):

$$
\begin{aligned}
S_{GR} &= \int d^4x\sqrt{-g}\left(\frac{c^4}{16\pi G}R + \mathcal{L}_m\right), &(3)\\
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \frac{16\pi G}{c^4}T_{\mu\nu} &(4)
\end{aligned}
$$

- GR has essentially passed all experimental/observational tests
- Nevertheless there are reasons to study modified gravity theories

Planck units

- We will use *(reduced) Planck units*: $8\pi G = c = \hbar = k_B = 1$
- Everything can be phrased in terms of the *geometrized dimension L*
- Energy scale, etc. are multiples of:
  - Planck energy: $E_p = l_p c^4/G \sim 10^{16} ergs \sim 10^{19} GeV$
  - Planck length: $l_p = (G\hbar/c^3)^{1/2} \sim 10^{-33} cm$
  - Planck time: $t_p = l_p/c \sim 10^{-44} s$
  - Planck mass: $m_p = l_p c^2/G \sim 10^{-5} g$
  - Planck temperature $E_p/k_B \sim 10^{32} K$

Outline

Review: motivations to study modified gravity theories

EdGB gravity

Approaches to studying modified gravity theories

Nonperturbative dynamics of EdGB gravity

Shift symmetric

$\mathbb{Z}_2$ symmetric

Conclusion

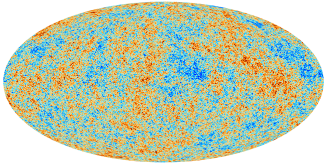
Why study modified gravity theories?

- Find a complete theory of quantum gravity
- Model the dynamics of the very early universe
- Model the dynamics of the late universe
- Model the interior of black holes
- Null Convergence Condition
- Test GR for sake of basic science

Find a complete theory of quantum gravity

- GR is *nonrenormalizable*: the gravitational coupling constant, G, has units of $(M_P)^2$ (M_P is the Planck mass.)
- Nonrenormalizability hints that GR could/'should' be modified at energies around the Planck scale $l_p \sim 10^{-33} cm$

Cosmology and GR



- At the largest scales the universe is approximately:
  1. homogeneous
  2. isotropic
  3. expanding
  4. Spatial sections are geometrically flat ($^{(3)}R_{ijkl} = 0$)
- Friedman-Lemaitre-Robertson-Walker (FLRW) solutions to the Einstein Equations
- With **suitable** matter contributions, the FLRW solutions match observational cosmological data extremely well

Early universe cosmology and GR: basic question

- What mechanism set the initial conditions for the universe?
  - *inflation*: introduce new fields/modify GR
  - *ekpyrosis*: introduce new fields/modify GR
- FLRW cosmologies are *geodesically incomplete*: what preceded the 'big bang'?

Late universe and GR

- To model the recent/late time expansion of the universe, need to add a *cosmological constant* Λ to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = T_{\mu\nu}. \tag{5}$$

- Is there a physical mechanism that sets the value of the cosmological constant, or is it a new 'fundamental' constant of nature?

Model the interior of black holes

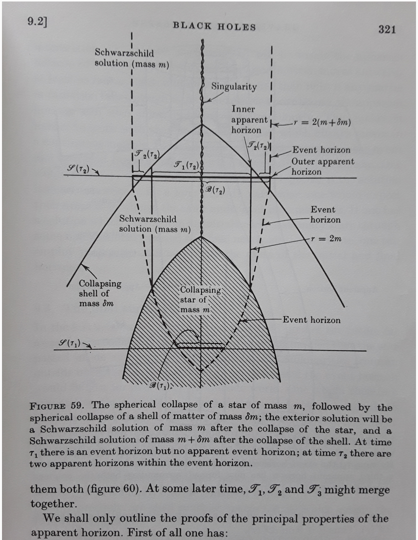

9.2] BLACK HOLES 321

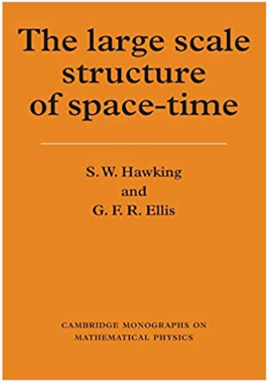
FIGURE 59. The spherical collapse of a star of mass m, followed by the spherical collapse of a shell of matter of mass δm; the exterior solution will be a Schwarzschild solution of mass m after the collapse of the star, and a Schwarzschild solution of mass $m+\delta m$ after the collapse of the shell. At time τ_1 there is an event horizon but no apparent event horizon; at time τ_2 there are two apparent horizons within the event horizon.

them both (figure 60). At some later time, $\mathscr{T}_1$, $\mathscr{T}_2$ and $\mathscr{T}_3$ might merge together.

We shall only outline the proofs of the principal properties of the apparent horizon. First of all one has:

- Inside black holes: spacetime solutions to GR are *incomplete*
- Unless GR is modified, an observer would reach the 'end' of space and time
- Cosmic censorship: the observer would be torn to pieces by tidal forces before reaching any 'singularity' or end to spacetime

Null Convergence Condition (NCC)

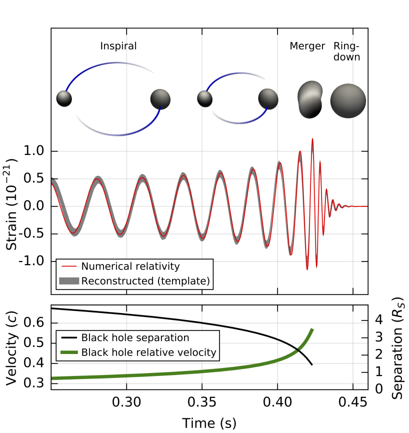


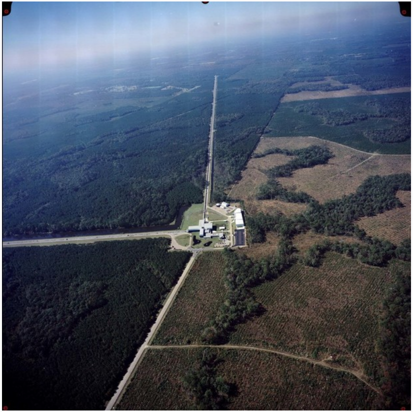
- For all null vectors k^a,

$$R_{ab}k^a k^b \geq 0 \tag{6}$$

- Plays important role in incompleteness (singularity) theorems
  - Incompleteness of FLRW cosmologies (big bang)
  - Incompleteness inside of black holes
- Standard classical matter fields cannot stably violate NCC
- Need to modify general relativity

Test GR for the sake of basic science: gravitational waves





- Gravitational potential of earth $\sim 10^{-9}$
- Employ *matched filtering* to extract gravitational wave signals: need to accurately model the physics!

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$$S_{EdGB} = \frac{1}{2} \int d^4 x \sqrt{-g} \left(R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V\left(\phi\right) - W\left(\phi\right) \mathcal{G} \right), \tag{7}$$

Gauss-Bonnet scalar

$$\mathcal{G} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\alpha\nu\beta} R^{\mu\alpha\nu\beta}. \tag{8}$$

Why EdGB gravity?

$$W(\phi)\,\mathcal{G}$$

- Find a complete theory of quantum gravity
  - Scalar Gauss-Bonnet term is order scalar-tensor mixing terms that may be found in the low energy limits of string theory
- Model the dynamics of the very early universe
- Model the dynamics of the late universe
- Model the interior of black holes
  - Scalar Gauss-Bonnet term shows up in some theories that attempt to address these problems
- Null Convergence Condition
  - EdGB gravity violates NCC for some solutions

Gravitational waves and testing GR

There are very few modified gravity theories that[1]

1. Are consistent with General Relativity in regimes where it is well tested
2. Predict observable deviations in the dynamical, strong field regime relevant to black hole mergers
3. Possess a classically well-posed initial value problem

[1] Lehner and Pretorius, Ann.Rev.Astron.Astrophys. 52 (2014) 661-694

Gravitational waves and testing GR

1. Are consistent with General Relativity in regimes where it is well tested
   - EdGB gravity not highly constrained by, e.g. binary pulsar tests[2]
2. Predict observable deviations in the dynamical, strong field regime relevant to black hole mergers
   - EdGB has scalarized black hole solutions, so it may predict large deviations from GR [3]
3. Possess a classically well-posed initial value problem
   - Equations of motion are second order so there may be initial data configurations for which this is true for EdGB gravity[4]

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[2] Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010

[3] e.g. Kanti, Mavromatos, Tamvakis, Winstanley, Phys.Rev. D54 (1996) 5049-5058;Sotiriou, Zhou Phys.Rev. D90 (2014) 124063

[4] e.g. Zwiebach, Phys.Lett. 156B (1985) 315-317

Outline

Approaches to studying modified gravity theories[6]

- Order reduction approach to solve the equations of motion of a modified gravity theory [5]
- Study exact (nonperturbative) solutions to particular modified gravity theories

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[5] can be used with effective field theories

[6] For further discussion, see e.g.
Cayuso, Ortiz, Lehner, Phys.Rev. D96 (2017) no.8, 084043;
Allwright, Lehner, Class.Quant.Grav. 36 (2019) no.8, 084001

Study nonperturbative effects

Directly solve full nonlinear set of equations (our approach)

$$S_{EdGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - W(\phi)\mathcal{G}\right), \quad (9)$$

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^\alpha\nabla_\gamma W(\phi)\right)\delta^\beta{}_{(\mu}g_{\nu)\delta} & \\ -\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 + \frac{1}{2}g_{\mu\nu}V(\phi) = 0, & \quad (10) \\ \Box\phi - V'(\phi) - W'(\phi)\mathcal{G} = 0. & \quad (11) \end{aligned}$$

Outline

Nonperturbative dynamics of EdGB gravity

Goal: To study the full equations of motion of a EdGB gravity
Challenge: We must then understand if theory has well posed initial value problem, obeys cosmic censorship, etc.
Reward: Could understand how modifications of GR affects the nonlinear dynamics of two black holes when they merge (and see how that affects the gravitational waveform produced during the merger)

Roadmap

- Study the dynamics of EdGB gravity in spherical symmetry for several choices of $V(\phi)$ and $W(\phi)$
  - Shift symmetric EdGB: $V(\phi) = 0$, $W(\phi) = \lambda\phi$
  - $\mathbb{Z}_2$ symmetric EdGB: $V(\phi) = m^2\phi^2$, $W(\phi) = \frac{1}{4}\eta\phi^2$
- Look at black hole solutions to the theory, and if for interesting features of the solutions
  - Scalarized black holes
  - Null Convergence Condition violation
  - Character of the equations of motion as partial differential equations

$$S_{EdGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - W(\phi)\mathcal{G}\right), \tag{12}$$

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Shift symmetric EdGB gravity

$$S_{EdGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - (\nabla\phi)^2 + 2\lambda\phi\mathcal{G}\right), \qquad (13)$$

Shift symmetric EdGB gravity: unique shift symmetric $\phi \to \phi + c$ scalar-tensor theory that does not admit **stationary** Schwarzschild black hole solutions[7]

[7] Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

Numerical setup

- Schwarzschild-like coordinates

$$ds^2 = - e^{2A(t,r)} dt^2 + e^{2B(t,r)} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) . \tag{14}$$

- Second order finite difference methods to solve the equations of motion

Equations of motion for EdGB gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + 2\lambda \delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma} R^{\rho\sigma}{}_{\kappa\lambda} \left(\nabla^{\alpha}\nabla_{\gamma}\phi\right) \delta^{\beta}{}_{(\mu} g_{\nu)\delta} - \nabla_{\mu}\phi\nabla_{\nu}\phi + \frac{1}{2} g_{\mu\nu} \left(\nabla\phi\right)^2 = 0, \qquad (15)$$

$$\Box\phi + \lambda\mathcal{G} = 0. \qquad (16)$$

Take algebraic combinations of equations of motion to get a hyperbolic (wave-like) equation for ϕ, and ordinary differential equations in r for the metric variables A and B

Results in Schwarzschild-like coordinates

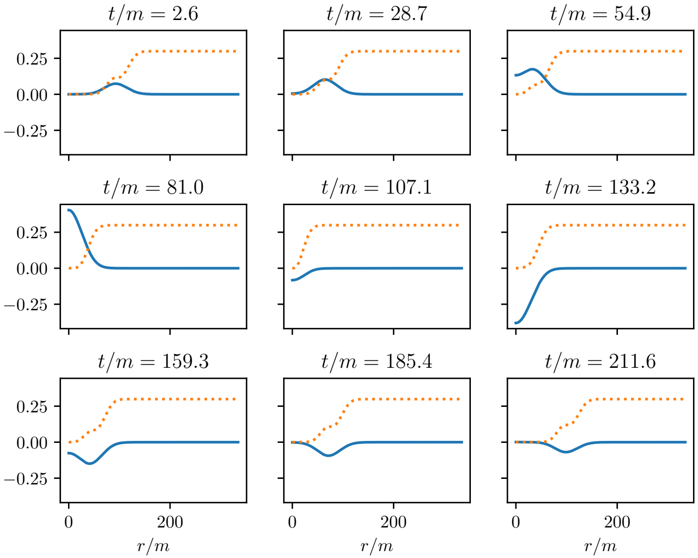
$$\begin{aligned}\mathcal{A}\partial_t^2\phi - \mathcal{B}\partial_t\partial_r\phi + \mathcal{C}\partial_r^2\phi + \cdots =& 0, & (17)\\ \partial_r A + \cdots =& 0, & (18)\\ \partial_r B + \cdots =& 0. & (19)\end{aligned}$$

- Study: small initial EdGB scalar pulse, with no initial black hole [8]
- Initial data

$$\phi(t,r)|_{t=0} = a_0 \left(\frac{r}{w_0}\right)^2 \exp\left(-\left(\frac{r-r_0}{w_0}\right)^2\right). \tag{20}$$

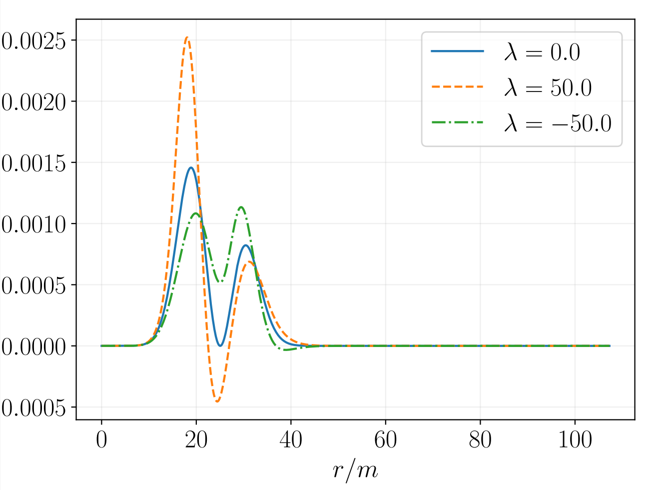
[8] JLR and Frans Pretorius Phys. Rev. D 99, 084014 (2019), JLR and Frans Pretorius Class.Quant.Grav. 36 (2019) no.13, 134001

Example scalar field evolution: scalar field disperses



Example scalar field evolution: null convergence condition

$$2\lambda\phi\mathcal{G} \qquad R_{\mu\nu}k^{\mu}k^{\nu} \tag{21}$$



Diagnostics: hyperbolicity of theory

- **Challenge:** must then understand if theory has well posed initial value problem, obeys cosmic censorship, etc.
- Well posed initial value problem: theory has a *strongly hyperbolic* formulation

Diagnostics: hyperbolicity

- Hyperbolicity: all the characteristic speeds in the theory are real
- Characteristic speeds: speed at which high frequency, linear perturbations travel about background solution
- Example:

$$h(x)\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} - V(x)f = 0. \tag{22}$$

Characteristic speeds are $c_{\pm} = \pm\sqrt{1/h(x)}$.

Characteristics

- Hyperbolicity: all the characteristic speeds in the theory are real
- Characteristic speeds: speed at which high frequency, linear perturbations travel about background solution
- Example:

$$h(x)\frac{\partial^2 f}{\partial t^2} - \frac{\partial^2 f}{\partial x^2} - V(x)f = 0. \tag{23}$$

Characteristic speeds are $c_\pm = \pm\sqrt{1/h(x)}$.

Characteristic speed

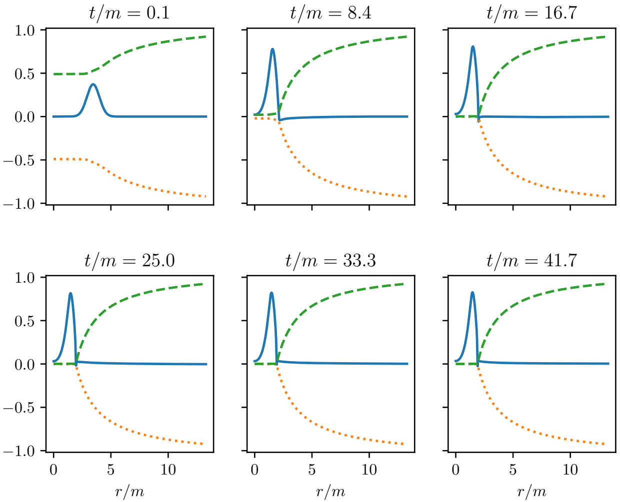
Diagnostics: hyperbolicity

- Hyperbolicity: all the characteristic speeds in the theory are real
- For EdGB gravity in spherical symmetry, characteristic speeds c obey equation of the form:

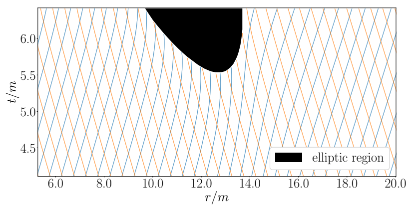
$$\mathcal{A}c^2 + \mathcal{B}c + \mathcal{C} = 0, \tag{24}$$

  where $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ depend on metric variables, scalar field, and their derivatives.
- Three regimes: parabolic, elliptic, and hyperbolic depending on sign of the discriminant $\mathcal{D} \equiv \mathcal{B}^2 - 4\mathcal{A}\mathcal{C}$.

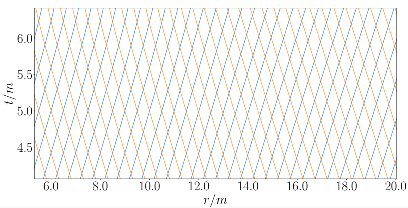
Scalar field evolution and characteristics for $\lambda R \ll 1$



EdGB gravity with $R\lambda \sim 0.1$

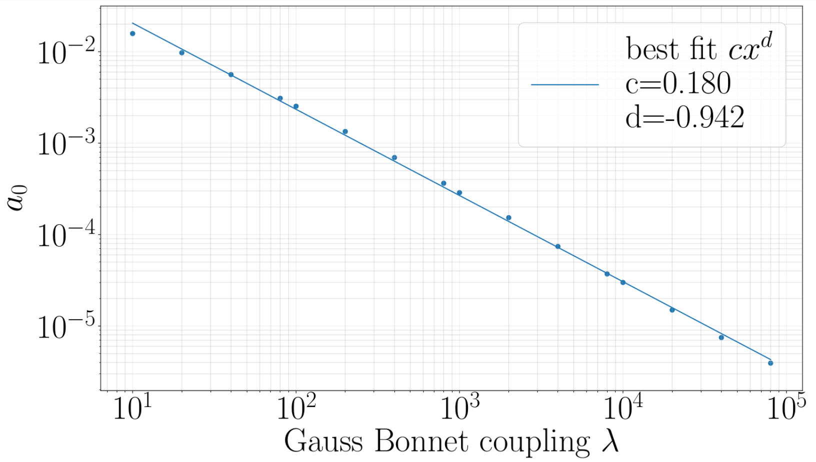


(a) EdGB characteristics



(b) Null characteristics

Elliptic vs hyperbolic evolution



$$\phi(t,r)|_{t=0} = a_0 \left(\frac{r}{w_0}\right)^2 \exp\left(-\left(\frac{r-r_0}{w_0}\right)^2\right). \tag{25}$$

EdGB gravity as "mixed-type" equations

- We find EdGB equations of motion for scalar field are hyperbolic up to a given curvature scale $R \times \lambda \sim 0.1$, then equations become *elliptic*
- Mixed type PDE: solution regions where elliptic, hyperbolic
- Example: Tricomi equation

$$\partial_x^2 f + x\partial_y^2 f = 0. \tag{26}$$

- Separation line between hyperbolic and elliptic part: **sonic line**

“Mixed-type” PDE

- Terminology comes from fluid dynamics: equation of motion for steady state solutions to inviscid, compressible flow obey a mixed-type equation[9]

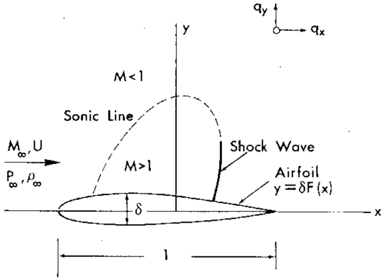


Fig. 1 Typical transonic flow past a slender airfoil.

Figure: J. D. Cole and E. M. Murman. *Calculation of Plane Steady Transonic Flow*

[9] e.g. C. Morawetz, *Mathematical Approach to the Sonic Barrier*

Simulations in Painlevé-Gullstrand (PG) coordinates

$$ds^2 = -\alpha(t,r)^2 dt^2 + (dr + \alpha(t,r)\zeta(t,r)dt)^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{27}$$

- Horizon penetrating
- Not singularity avoiding
- spatially flat: ${}^{(3)}R_{ijkl} = 0$

$$\alpha = 1, \qquad \zeta = \sqrt{\frac{2m}{r}}. \tag{28}$$

Equations of motion in PG coordinates for EdGB gravity

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^{\alpha}\nabla_{\gamma}\phi\right)\delta^{\beta}{}_{(\mu}g_{\nu)\delta}$$

$$-\nabla_{\mu}\phi\nabla_{\nu}\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 = 0, \qquad (29)$$

$$\Box\phi + \lambda\mathcal{G} = 0. \qquad (30)$$

- Through taking algebraic combinations of the equations of motion, can define wave-like equation for ϕ (with no time derivatives acting on α and ζ).
- Hamiltonian and momentum constraints give ordinary differential equations (in r) for metric fields α and ζ

Other numerical tools

- Solve PDE/ODE system with (second order) finite difference methods
- Spatial compactification ($x = L$ is spatial infinity)

$$r(x) \equiv \frac{x}{1 - x/L} \tag{31}$$

- Modified Berger-Oliger style fixed mesh refinement to solve system of hyperbolic and elliptic (ode) equations[10]

[10] Pretorius and Choptuik, J.Comput.Phys. 218 (2006) 246-274

Schwarzschild initial data

- Schwarzschild initial data
- Curvature coupling with black hole mass m

$$C \equiv \frac{\lambda}{m^2}. \tag{32}$$

- Compare to "decoupled" scalar field solution: time independent solution of Schwarzschild background with scalar field obeying

$$\Box\phi + \lambda\mathcal{G} = 0. \tag{33}$$

Growth of scalar hair

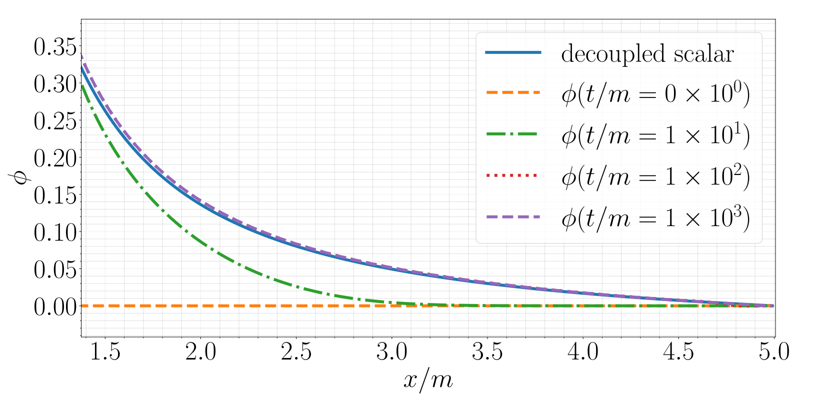
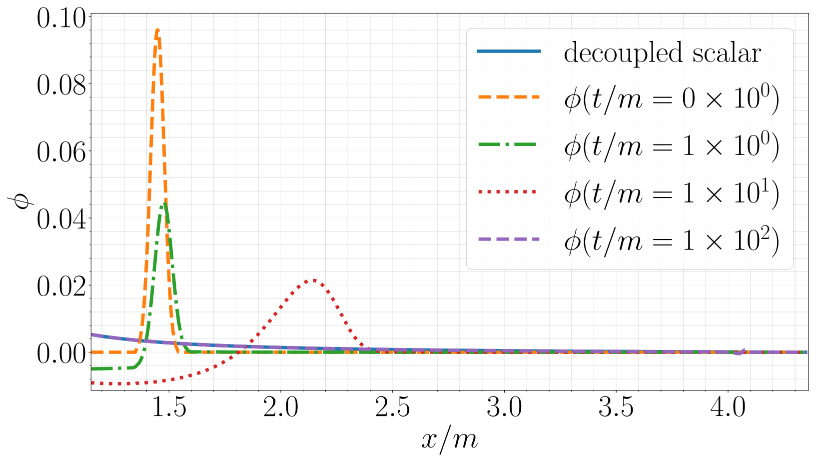


Figure: $C = 0.16$. The horizon (MOTS) is located at $x_h/m \approx 1.48$, and spatial infinity is at $x/m = 5$.

Schwarzschild initial data with a perturbation

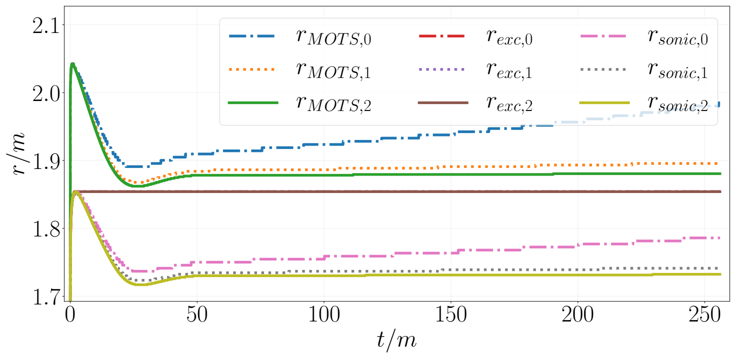


$$\phi(t,r)\big|_{t=0} = \begin{cases} \phi_0 \exp\left[-\frac{1}{(r-a)(b-r)}\right] \exp\left[-5\left(\frac{r-(a+b)/2}{a+b}\right)^2\right] & a < r < b \\ 0 & \text{otherwise} \end{cases}$$

Sonic line and black holes in EdGB gravity

- Sonic line forms inside EdGB black hole for any (nonzero) value of λ
- Geometry is smooth and finite up to sonic line (cannot say what geometry is past sonic line)
- For small enough λ/m^2, sonic line inside black hole horizon and elliptic region is "censored"
- For large enough λ/m^2, sonic line outside black hole horizon

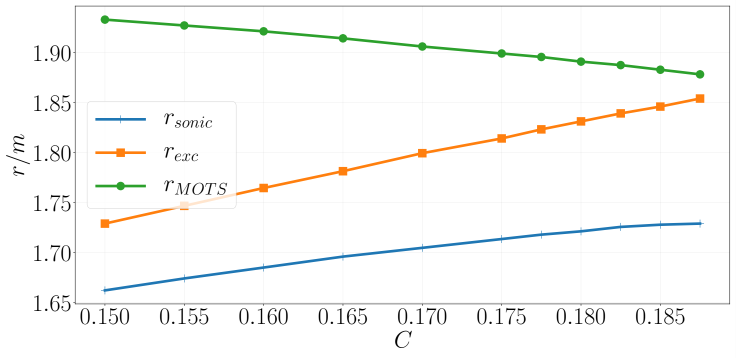
Evolution of sonic line vs. apparent horizon



EdGB black holes: elliptic region inside of black hole for small enough curvature couplings

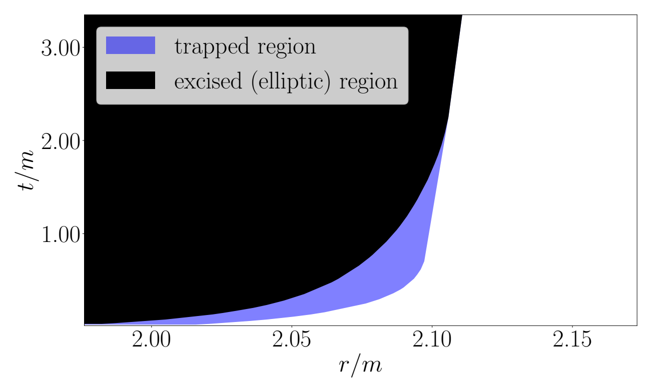
Evolution of sonic line vs. apparent horizon

Sonic point approaches black hole horizon for larger curvature couplings (estimate "extremal" $\lambda/m^2 \sim 0.23$)



“Superextremal” curvature-coupling

For large enough curvature couplings, elliptic region grows outside of black hole: have “naked” elliptic region and can on longer run simulations



“Superextremal” curvature-coupling

- Formation of naked elliptic region: breakdown of casual evolution
- No universal curvature coupling value for which have “naked” elliptic region
- With large enough gradients can trigger elliptic region formation, so triggering elliptic region formation depends on initial data [11]

[11] JLR and Frans Pretorius Phys. Rev. D 99, 084014 (2019), JLR and Frans Pretorius Class.Quant.Grav. 36 (2019) no.13, 134001

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$\mathbb{Z}_2$ symmetric EdGB[12]

$$S_{\mathbb{Z}_2} = \int d^4x\sqrt{-g}\left(R - (\nabla\phi)^2 - \mu^2\phi^2 - 2\lambda\phi^4 + \frac{1}{8}\eta\phi^2\mathcal{G}\right). \quad (34)$$

- symmetric under $\phi \to -\phi$
- Scalarized black holes can form in this theory depending on value of μ, λ, η, and black hole mass

[12] Macedo et. al., Phys.Rev. D99 (2019) no.10, 104041

$\mathbb{Z}_2$ symmetric EdGB[13]

- With black holes of mass M define dimensionless quantities

$$\hat{M} \equiv \frac{M}{\eta^{1/2}}, \tag{35}$$
$$\hat{\mu} \equiv \mu\eta^{1/2}, \tag{36}$$
$$\hat{\lambda} \equiv \eta\lambda. \tag{37}$$

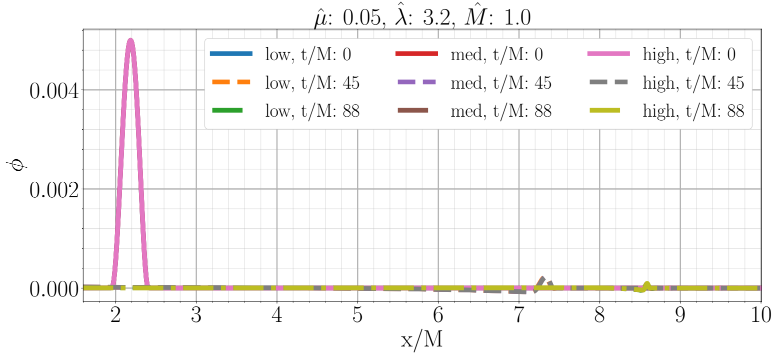
- For large enough $\hat{\lambda}$ and small enough $\hat{M}$ radially stable scalarized black holes exist in this theory
- For small enough $\hat{M}$ an elliptic region can form outside of the black hole horizon
- For large enough $\hat{\lambda}$, there is a band of $\hat{M}$ values that lead to scalarized black holes but no naked elliptic region
- With nonzero $\hat{\lambda}$, for scalarized black holes there is an elliptic region inside the horizon

[13] Macedo et. al., Phys.Rev. D99 (2019) no.10, 104041

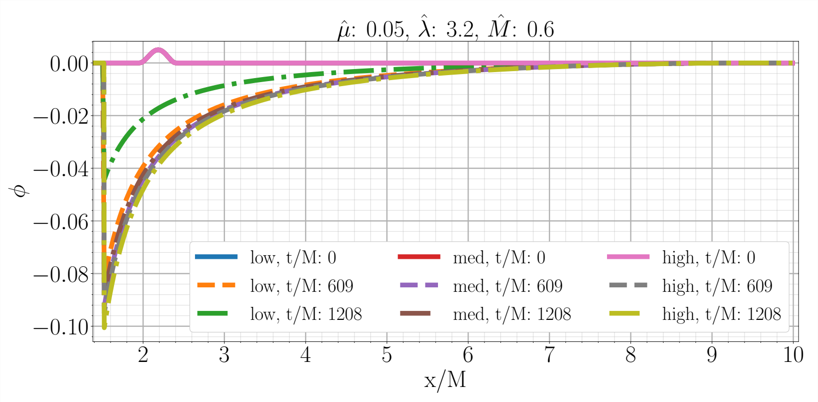
Numerical setup

- Work in Painlevé-Gullstrand coordinates
- Spatially compactified domain
- Unigrid evolution
- Always have black hole in initial data, with nonzero scalar field profile outside of black hole

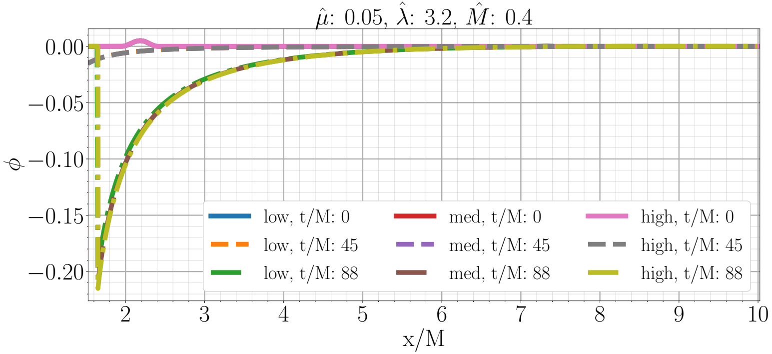
Example: scalar field disperses and stable Schwarzschild black hole results



Example: black hole scalarization



Example: naked elliptic region forms



Outline

Publications related to this presentation

- JLR and Frans Pretorius, Phys.Rev.D 99 (2019) 8, 084014, arXiv:1902.01468
- JLR and Frans Pretorius, Class.Quant.Grav. 36 (2019) 13, 134001, arXiv:1903.07543
- JLR, Class.Quant.Grav. 36 (2019) 23, 237001, arXiv:1908.04234
- JLR and Frans Pretorius, Phys.Rev.D 101 (2020) 4, 044015, arXiv:1911.11027
- JLR and Frans Pretorius, Class.Quant.Grav, arXiv:2005.05417

Future directions

- Study other modified gravity theories in spherical symmetry and look at NCC, hyperbolicity, etc
- Kovacs and Reall: 'modified harmonic gauge'[14] that allows for solving EdGB gravity (and other 'Horndeski theories') in general backgrounds
- Implement this gauge with a version of Frans' generalized harmonic code
- Ultimate goal: black hole simulations and gravitational wave signals

[14] Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

Conclusion

- GR is an extremely successful theory of gravity, but there are still reasons to study modified gravity theories
- Study of the nonlinear dynamics of EdGB gravity in spherical symmetry
  - Scalarized black holes can form for some versions of the theory and for some parameter ranges
  - EdGB gravity violates weak/strong cosmic censorship
- potential future directions:
  - Study EdGB gravity in less symmetry restricted spacetime in modified harmonic gauge
  - Apply analysis to other varieties of EdGB gravity, or other modified gravity theories

Additional work on well-posedness of modified gravity theories

- Bernard, Lehner, and Luna[15] consider spherically symmetric dynamics of

$$\mathcal{L} = (1 + G_4(\phi))\, R + (\partial\phi)^2 - V(\phi) + G_2\left(\phi, (\partial\phi)^2\right). \quad (38)$$

  see also Papallo and Reall, who study Horndeski theories in less symmetric spacetimes[16].

- Kovacs and Reall[17] have found a set of gauge conditions that may lead to well-posed evolution for small enough deviations from GR

$$\mathcal{L} = R + (\partial\phi)^2 + f_1(\phi)\,(\partial\phi)^4 - V(\phi) + f_2(\phi)\mathcal{G}. \quad (39)$$

[15] Phys.Rev. D100 (2019) no.2, 024011
[16] Papallo, Reall, Phys.Rev. D96 (2017) no.4, 044019
[17] 2003.08398

Shift symmetric EdGB gravity in these two approaches

$$S_{EdGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\lambda\phi\mathcal{G}\right), \tag{40}$$

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^\alpha\nabla_\gamma\phi\right)\delta^\beta{}_{(\mu}g_{\nu)\delta} \\ -\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 = 0, \end{aligned} \tag{41}$$

$$\Box\phi + \lambda\mathcal{G} = 0. \tag{42}$$

Order reduction approach for EdGB gravity[18]

Assume $\epsilon \sim \lambda$ and $|\epsilon| \ll 1$

$$
\begin{aligned}
g_{\mu\nu} =& g^{(0)}_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + \cdots \\
\phi =& \phi^{(0)} + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots
\end{aligned}
\tag{43a}
$$

$$
\phi^{(0)} = 0, \tag{44a}
$$

$$
R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{44b}
$$

$$
\Box \phi^{(1)} = \lambda \mathcal{G}\left[g^{(0)}_{\alpha\beta}\right], \tag{45a}
$$

$$
R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{45b}
$$

$$
R_{\mu\nu}[g^{(2)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(2)}_{\alpha\beta}] = \lambda \times F\left[\phi^{(1)}\right] \tag{46}
$$

[18] Okounkova, *Phys. Rev. D* 100 (2019)