The classical evolution of binary black hole systems in scalar-tensor theories[1]

Seminar, University of Oxford

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with William E. East

DAMTP, University of Cambridge

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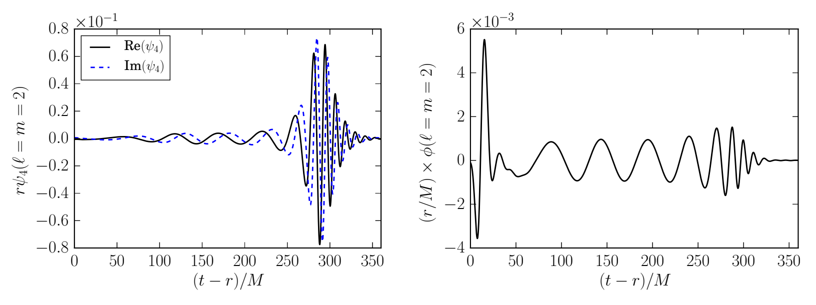
[1] Mostly based on arXiv:2011.03547

Outline and Summary

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left(R + X - V(\phi) + \alpha(\phi)X^2 + \beta(\phi)\mathcal{G}\right),$$

$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \qquad \mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}$$



Goals: understand why we choose to study the above theory, and understand how we made these plots!

Outline and Summary

- Why study scalar-tensor gravity theories?
- Generating gravitational waveforms for scalar-tensor gravity theories
- Technical/mathematical advances that made this possible (if there is time/interest)

Planck units

- We will use *(reduced) Planck units*: $8\pi G = c = \hbar = k_B = 1$
- Everything can be phrased in terms of the *geometrized dimension L*
- Energy scale, etc. are multiples of:
  - Planck energy: $E_p = l_p c^4/G \sim 10^{16} ergs \sim 10^{19} GeV$
  - Planck length: $l_p = (G\hbar/c^3)^{1/2} \sim 10^{-33} cm$
  - Planck time: $t_p = l_p/c \sim 10^{-44} s$
  - Planck mass: $m_p = l_p c^2/G \sim 10^{-5} g$
  - Planck temperature $E_p/k_B \sim 10^{32} K$

Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

Scalar-tensor (Horndeski) gravity

Theories that have a tensor ($g_{\mu\nu}$) field and scalar (ϕ) field, and have second order equations of motion

$$
\begin{aligned}
S =& \int d^4x\sqrt{-g}\left(\mathcal{L}_1+\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5\right),\\
\mathcal{L}_1 \equiv& \frac{1}{2}R + X - V(\phi),\\
\mathcal{L}_2 \equiv& G_2\left(\phi, X\right),\\
\mathcal{L}_3 \equiv& G_3\left(\phi, X\right)\Box\phi,\\
\mathcal{L}_4 \equiv& G_4\left(\phi, X\right)R + \partial_X G_4\left(\phi, X\right)\delta^{\mu\nu}_{\alpha\beta}\nabla^\alpha\nabla_\mu\phi\nabla^\beta\nabla_\nu\phi,\\
\mathcal{L}_5 \equiv& G_5\left(\phi, X\right)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\partial_X G_5\left(\phi, X\right)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla^\beta\phi\nabla_\rho\nabla^\gamma\phi,\\
X \equiv& -\frac{1}{2}\left(\nabla\phi\right)^2,
\end{aligned}
$$

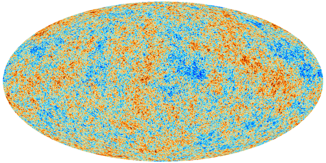
Why study scalar-tensor gravity?

- Find a complete theory of quantum gravity
- Model the dynamics of the very early universe
- Model the dynamics of the late universe
- Test GR for sake of basic science

Find a complete theory of quantum gravity

- GR is *nonrenormalizable*: the gravitational coupling constant, G, has units of $(M_P)^2$ (M_P is the Planck mass.)
- Nonrenormalizability hints that GR could/'should' be modified at energies around the Planck scale $l_p \sim 10^{-33} cm$

Cosmology and GR



- At the largest scales the universe is approximately:
  1. homogeneous
  2. isotropic
  3. expanding
  4. Spatial sections are geometrically flat ($^{(3)}R_{ijkl} = 0$)
- Friedman-Lemaitre-Robertson-Walker (FLRW) solutions to the Einstein Equations
- With suitable matter contributions and a cosmological constant, the FLRW solutions match observational cosmological data extremely well

Late universe and GR

- To model the recent/late time expansion of the universe, need to add a *cosmological constant* Λ to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = T_{\mu\nu}.$$

- Is there a physical mechanism that sets the value of the cosmological constant, or is it a new fundamental constant of nature?

Late universe and GR

- If you want to have "super-accelerated" expansion, where expansion happens *faster* than is possible with a cosmological constant (i.e. when the effective equation of state $w < -1$), then typically you need to modify gravity with higher derivative terms[2]

THE GALILEON AS A LOCAL MODIFICATION OF GRAVITY

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[2] e.g. Phys.Rev.D 79 (2009) 064036 arXiv:0811.2197 [hep-th]

Early universe cosmology and GR: basic questions

- What mechanism set the initial conditions for the universe?[3]
- FLRW cosmologies are *geodesically incomplete*: what preceded the 'big bang'?

Generalized G-inflation: Inflation with the most general second-order field equations

Tsutomu Kobayashi[a,b], Masahide Yamaguchi[c], and Jun'ichi Yokoyama[d,e]

[a] *Hakubi Center, Kyoto University, Kyoto 606-8302, Japan*
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Galilean Genesis: an alternative to inflation

Paolo Creminelli[a], Alberto Nicolis[b] and Enrico Trincherini[c,d]

[a] *Abdus Salam International Centre for Theoretical Physics Strada Costiera 11, 34151, Trieste, Italy*

Fully stable cosmological solutions with a non-singular classical bounce

CrossMark

Anna Ijjas[a,*], Paul J. Steinhardt[a,b]

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ARTICLE INFO ABSTRACT

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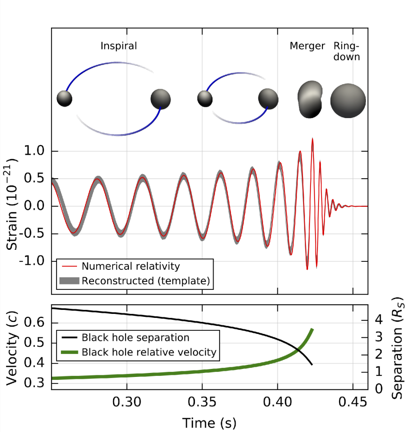
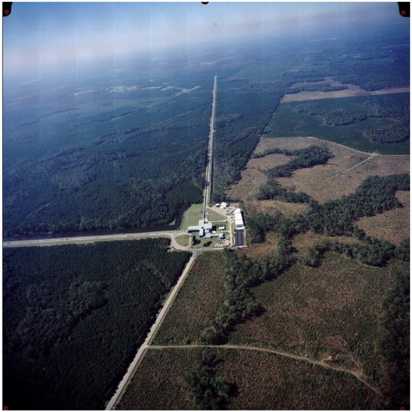
[3] references to above papers: Prog.Theor.Phys. 126 (2011) 511-529, arXiv:1105.5723; JCAP 11 (2010) 021, arXiv:1107.0027; Phys.Lett.B 764 (2017) 289-294, arXiv:1609.01253

Test GR for the sake of basic science: gravitational waves





- Gravitational potential of earth $\sim 10^{-9}$
- Employ *matched filtering* to extract gravitational wave signals: need to accurately model the physics!

Test GR with gravitational waves: the need for accurate source modeling

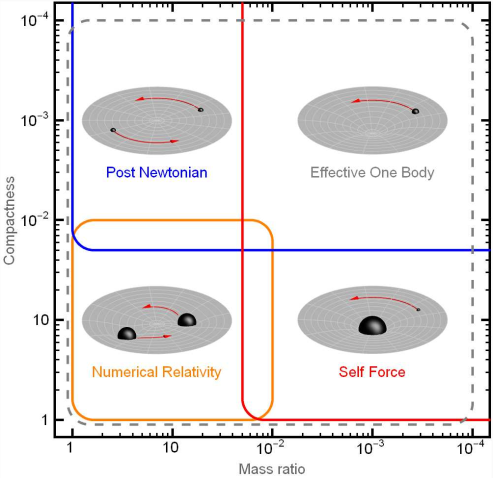


Figure: https://en.wikipedia.org/wiki/Two-body_problem_in_general_relativity

Guiding principles

Can we find a classical field theory that

1. Has a mathematically sensible interpretation?
2. Matches all current observations?
3. Addresses a current problem in physics?
   3.1 Renormalizable (or leading order interactions of a sensible quantum theory of gravity)?
   3.2 Incompleteness of early universe or black holes (and so admits NCC violating solutions)?
4. Can be tested/constrained with new observations?

Outline

sEFT gravity

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left(R + X - V\left(\phi\right) + \alpha\left(\phi\right)X^2 + \beta\left(\phi\right)\mathcal{G}\right),$$

where

$$X \equiv -\frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi,$$

$\mathcal{G}$: the *Gauss-Bonnet scalar*

$$\mathcal{G} \equiv R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\alpha\nu\beta}R^{\mu\alpha\nu\beta}.$$

Why sEFT gravity?

1. Has a mathematically sensible interpretation?
   - Yes, provided the modified gravity corrections are "small"[4]
2. Matches all current observations?
   - Yes, provided we do not use this theory to model the late universe ESGB gravity not highly constrained by, e.g. binary pulsar tests[5]

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[4] e.g. JLR & Pretorius, Class.Quant.Grav. 36 (2019) 13, 134001, Kovacs et. al. Phys.Rev.D 101 (2020) 12, 1240030

[5] e.g. Baker et. al. Phys.Rev.Lett. 119 (2017) 25, 251301, Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010

Why sEFT gravity?

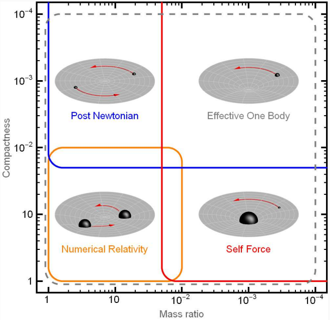
1. Addresses a current problem in physics?
   - Theory captures leading order scalar-tensor parity invariant interactions, so captures the leading order corrections from many UV complete theories of gravity[6]
2. Can be tested/constrained with new observations?
   - Many versions of the theory have ‘scalarized’ black hole solutions, so will be strongly constrained by gravitational wave observations[7]

[6] e.g. Weinberg, Phys.Rev.D 77 (2008) 123541

[7] e.g. Kanti et. al. Phys.Rev.D 54 (1996) 5049-5058

Approaches to studying modified gravity theories[9]

- Order reduction approach to solve the equations of motion of a modified gravity theory [8]
- **Study exact (nonperturbative) solutions to particular modified gravity theories: useful for understanding physics in strong field, dynamical regime**



[8] e.g. Okounkova etl al., Class.Quant.Grav. 36 (2019) 5, 054001; Okounkova et. al., Phys.Rev.D 99 (2019) 4, 044019

[9] e.g. Cayuso, Ortiz, Lehner, Phys.Rev. D96 (2017) no.8, 084043; Allwright, Lehner, Class.Quant.Grav. 36 (2019) no.8, 084001

Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

Addresses a current problem in physics?

- Theory captures leading order scalar-tensor parity invariant interactions, so captures the leading order corrections from many UV complete theories of gravity[10]

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left(R + X - V\left(\phi\right) + \alpha\left(\phi\right)X^2 + \beta\left(\phi\right)\mathcal{G}\right),$$

[10] e.g. Weinberg, Phys.Rev.D 77 (2008) 123541

Shift symmetric effective field theory ($\phi \to \phi + \mathit{const.}$)

- If you want to capture a theory that is invariant under shifts in ϕ (e.g. some classes of inflation theories)

$$S = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\left(R + X + \alpha_0 X^2 + \beta_0\phi\mathcal{G}\right),$$

- We will set $\alpha_0 = 0$, call $\beta_0 = \lambda$ (to match the notation of earlier studies in the literature)
- While setting $\alpha_0 = 0$ isn't well motivated from the standpoint of effective field theory, it simplifies studying the theory as we are only considering adding one new constant to the equations of motion

Shift symmetric ESGB gravity

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 2\lambda\phi\mathcal{G}\right),$$

- This theory does not admit **stationary** Schwarzschild black hole solutions[11]; instead "hairy" scalar black holes should be end states in this theory

$$\Box\phi + \lambda\mathcal{G} = 0 ,$$

[11] Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

Shift symmetric ESGB in a modified harmonic formulation[12]

- Collaboration with Will East
- Reformulate the equations of motion in *modified generalized harmonic* formulation
- Consider spinning black hole evolution (axisymmetric spacetime)
- Consider head on black hole collisions (axisymmetric spacetime)
- Consider binary black hole merger (no symmetry assumptions)

[12] arXiv:2011.03547

Modified generalized harmonic (MGH) formulation[13]

- Specify two auxiliary Lorentzian metrics $\hat{g}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ in addition to the spacetime metric $g^{\mu\nu}$
- Specify the gauge/coordinate condition with:

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu x^\gamma = H^\gamma, \tag{1}$$

  where H^γ is source function
- Free parameters: $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$, H^γ (more details given at end of talk)
- Besides using the MGH formulation, we begin with GR initial data, and use standard techniques from numerical relativity

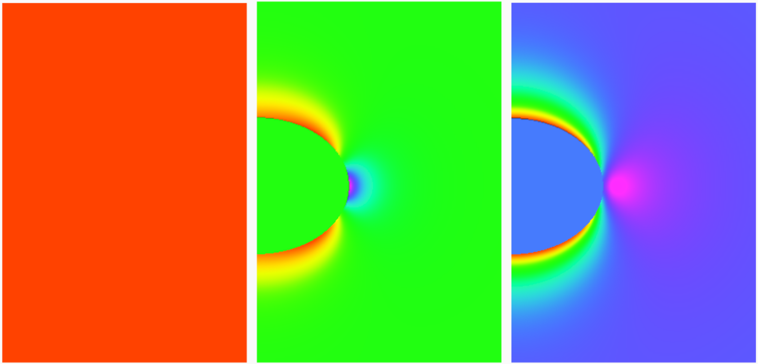
[13] Kovacs and Reall, Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

Initial conditions

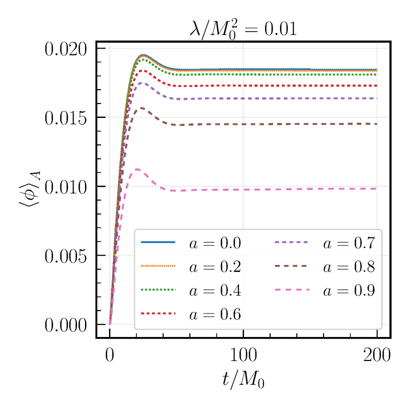
- For technical reasons, we always start with a GR solution (e.g. one spinning black hole, two boosted black holes), and then let the black holes grow scalar hair as we evolve in time
- After a finite amount of evolution, the black holes stop growing scalar hair (growth saturates)

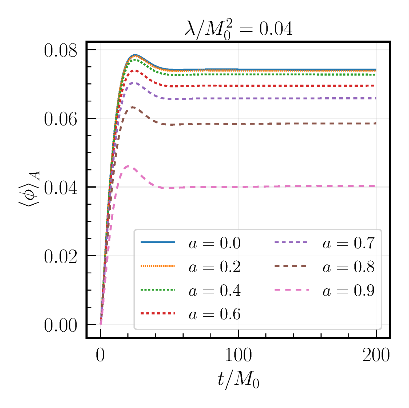
$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2\lambda\phi\mathcal{G}\right),$$

Scalar hair growth around spinning black holes



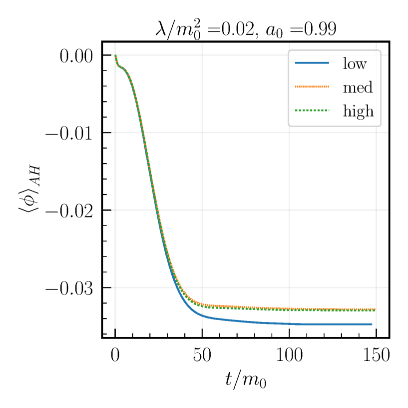
Scalar hair growth around spinning black holes





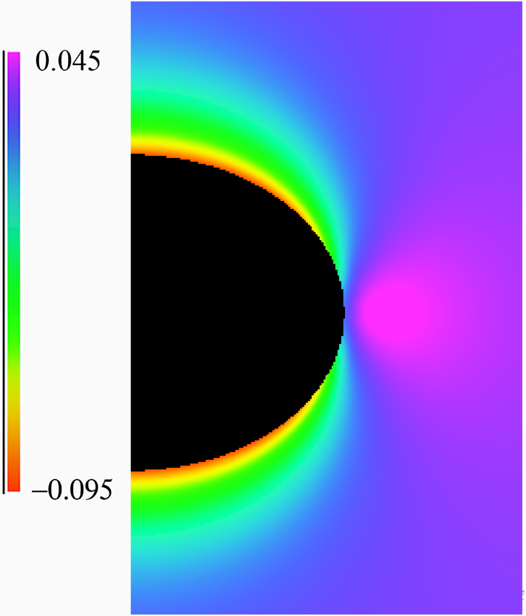
- $\langle\phi\rangle_A$: average scalar field value on black hole horizon
- a: initial dimensionless black hole spin

Scalar hair growth around spinning black holes

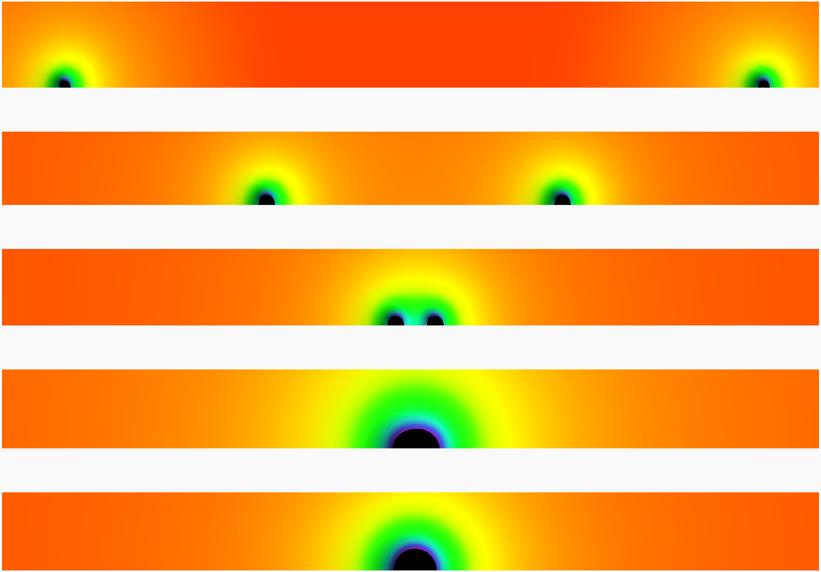


- $\langle \phi \rangle_A$: average scalar field value on black hole horizon, at three different resolutions (convergence study)

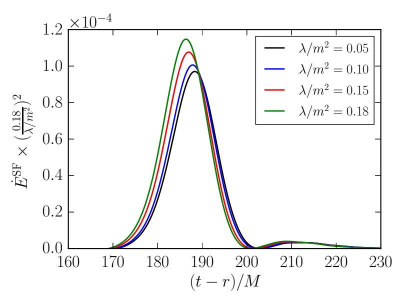
Scalar field density around a spinning black hole

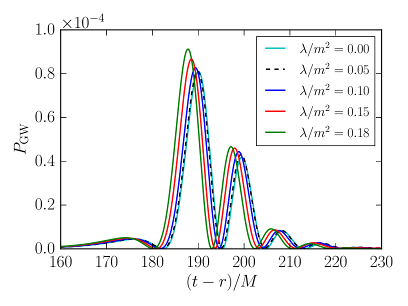


Head on black hole collisions



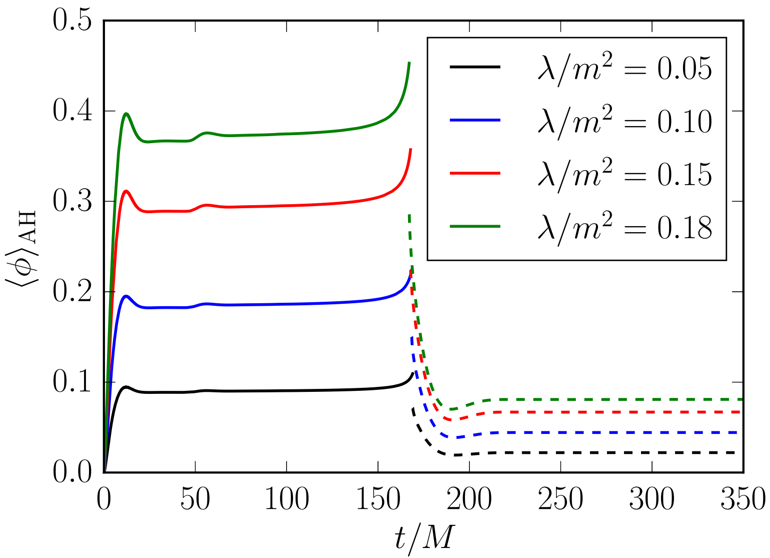
Head on black hole collisions: gravitational and scalar radiation



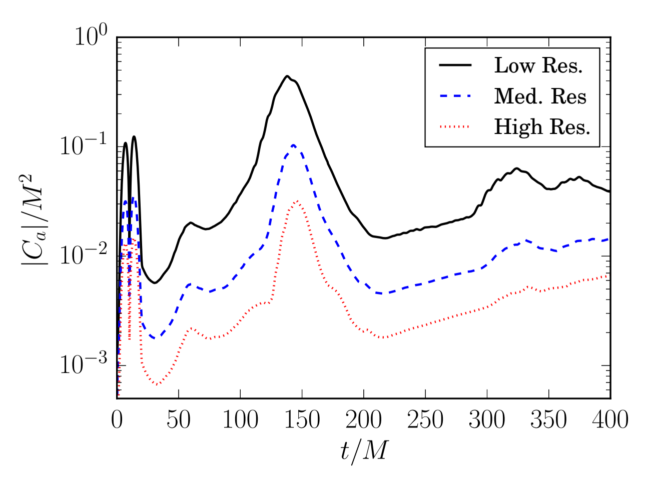


Flux of scalar field vs flux of gravitational waves

Head on black hole collisions: scalar field on horizon



Head on black hole collisions: convergence



Convergence of "constraint violation":

$$C^\alpha \equiv H^\alpha + \tilde{g}^{\mu\nu}\Gamma^\alpha_{\mu\nu} \quad (2)$$

Binary black hole collisions

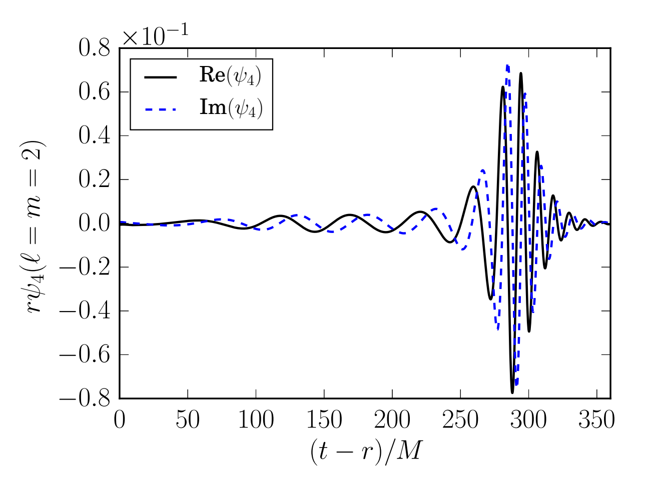


Figure: $\lambda/M^2 = 0.01$

Gravitational wave strain from two ESGB binary black holes merging

Binary black hole collisions

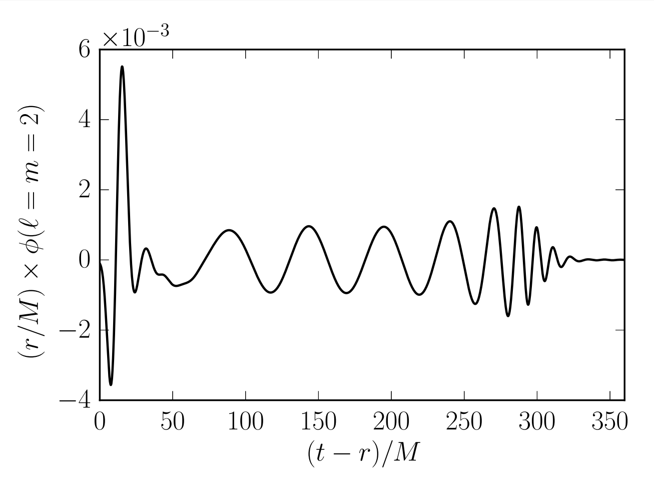


Figure: $\lambda/M^2 = 0.01$

What was the main challenge? Finding a well-posed initial value formulation for the theory

- sEFT gravity has a well-posed initial value problem in generic spacetimes, provided the modified gravity corrections are "small", when one specifies their coordinate according to a *modified generalized harmonic* (MGH) condition[14]:

[14] Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

Outline

Conclusion

- GR is an extremely successful theory of gravity, but there are still reasons to study modified gravity theories
  - early universe: inflation, genesis, bouncing, ...
  - late universe: dark energy, ...
- Can test GR with gravitational waves
  - for that you need gravitational waveform templates to compare to data
- **Claim:** We now have the tools to produce gravitational waveforms produced during the merger of two black holes for a whole class of scalar-tensor gravity theories

Future directions

- Further develop the MGH formulation of general relativity and scalar-tensor gravity theories
  - What are “good” choices for the auxiliary metrics?
  - Make contact with the BSSN-type formulations
- Evolution of other Horndeski gravity theories
  - Binary black hole waveform catalogues for other kinds of scalar-tensor gravity theories
  - Consider early universe cosmological simulations in these theories
- More systematic study of the binary black hole problem in MGH formulation
  - Better initial data
  - Compare waveforms of GR vs. modified gravity theories

More slides on the MGH formulation and initial value problem

Shift symmetric ESGB: equations of motion

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2\lambda\phi\mathcal{G}\right),$$

$$\begin{aligned} E^{(g)}_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^\alpha\nabla_\gamma\phi\right)\delta^\beta{}_{(\mu}g_{\nu)\delta} & \\ -\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 &= 0, \\ E^{(\phi)} \equiv \Box\phi + \lambda\mathcal{G} &= 0. \end{aligned}$$

Well-posed initial value problem: strongly hyperbolic formuation

$$\partial_t \mathsf{v} + \sum_{i=1}^{3} \hat{A}^i \partial_{x^i} \mathsf{v} + \mathsf{F} = 0.$$

- **Strongly hyperbolic:** Matrix $\hat{A}$ has real eigenvalues, and has a complete set of eigenvectors[15]
- Hyperbolicity of Einstein equations and Horndeski equations depends on the **formulation** of the equations of motion

[15] More technically, has a symmetrizer that one can bound independently of the derivatives of v; e.g. Sarbach and Tiglio, Living Rev.Rel. 15 (2012) 9

Why does the MGH formulation work?[16]

$$\partial_t \mathsf{v} + \sum_{i=1}^{3} \hat{A}^i \partial_{x^i} \mathsf{v} + \mathsf{F} = 0.$$

- In generalized harmonic formulation, $\hat{A}^i$ has all real eigenvalues (light speed): for GR the symbol can be diagonalized, but when adding modified gravity terms the matrix forms Jordan blocks due to ϕ, $g_{\mu\nu}$ couplings in the principal part
- In MGH formulation, eigenvalues have different values (depending on $g^{\mu\nu}$, $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$); a matrix with different real eigenvalues is more robust to remaining diagonalizable when adding modified gravity "perturbations" to the principal symbol

[16] For more discussion, see Kovacs and Reall, Phys. Rev. D 101, 124003 (2020)

More on the Modified generalized harmonic (MGH) formulation[17]

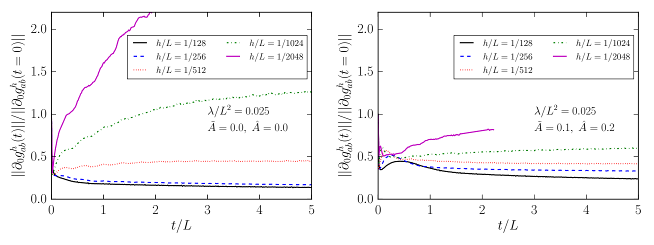
$$
\begin{aligned}
C^{\gamma} &\equiv H^{\gamma} - \tilde{g}^{\alpha\beta}\nabla_{\alpha}\nabla_{\beta}x^{\gamma} \\
&= H^{\gamma} + \tilde{g}^{\alpha\beta}\Gamma^{\gamma}_{\alpha\beta} = 0, \\
E^{\alpha\beta} - \hat{P}_{\delta}{}^{\gamma\alpha\beta}\nabla_{\gamma}C^{\delta} - \frac{1}{2}\kappa\left(n^{\alpha}C^{\beta} + n^{\beta}C^{\alpha} + \rho n^{\gamma}C_{\gamma}g^{\alpha\beta}\right) &= 0, \\
\hat{P}_{\delta}{}^{\gamma\alpha\beta} \equiv \frac{1}{2}\left(\delta^{\alpha}_{\delta}\hat{g}^{\beta\gamma} + \delta^{\beta}_{\delta}\hat{g}^{\alpha\gamma} - \delta^{\gamma}_{\delta}\hat{g}^{\alpha\beta}\right)&.
\end{aligned}
$$

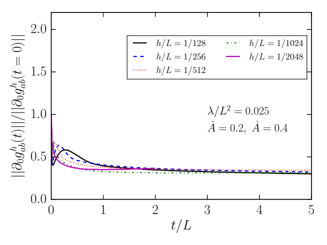
Divergence of equations of motion and use $\nabla_{\alpha}E^{\alpha\beta} = 0$, get propogation of constraint violation:

$$
-\frac{1}{2}\hat{g}^{\alpha\gamma}\nabla_{\alpha}\nabla_{\gamma}C^{\beta} - \hat{g}^{\gamma\beta}R_{\delta\gamma}C^{\delta} - \cdots = 0.
$$

[17] Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

Hyperbolicity test: Self-convergence in harmonic vs modified harmonic gauge





Order reduction approach for ESGB gravity[18]

Assume $\epsilon \sim \lambda$ and $|\epsilon| \ll 1$

$$\begin{aligned} g_{\mu\nu} =& g^{(0)}_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + \cdots \\ \phi =& \phi^{(0)} + \epsilon\phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots \end{aligned} \tag{3a}$$

$$\phi^{(0)} = 0, \tag{4a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{4b}$$

$$\Box\phi^{(1)} = \lambda\mathcal{G}\left[g^{(0)}_{\alpha\beta}\right], \tag{5a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{5b}$$

$$R_{\mu\nu}[g^{(2)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(2)}_{\alpha\beta}] = \lambda \times F\left[\phi^{(1)}\right] \tag{6}$$

[18] Okounkova, *Phys. Rev. D* 100 (2019)